Optical evaluation of the wave filtering properties of graded undulated lattices

G. Trainiti,1,a) J. J. Rimoli,1 and M. Ruzzene1,2
1Georgia Institute of Technology, Daniel Guggenheim School of Aerospace Engineering, Atlanta, Georgia 30332, USA
2Georgia Institute of Technology, George W. Woodruff School of Mechanical Engineering, Atlanta, Georgia 30332, USA

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We investigate and experimentally demonstrate the elastic wave filtering properties of graded undulated lattices. Square reticulates composed of curved beams are characterized by graded mechanical properties which result from the spatial modulation of the curvature parameter. Among such properties, the progressive formation of frequency bandgaps leads to strong wave attenuation over a broad frequency range. The experimental investigation of wave transmission and the detection of full wavefields effectively illustrate this behavior. Transmission measurements are conducted using a scanning laser Doppler vibrometer, while a dedicated digital image correlation procedure is implemented to capture in-plane wave motion at selected frequencies. The presented results illustrate the broadband attenuation characteristics resulting from spatial grading of the lattice curvature, whose in-depth investigation is enabled by the presented experimental procedures.

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I. INTRODUCTION

Structural lattices, obtained tessellating 2D and 3D space with slender beam elements, can be regarded as a special class of mechanical metamaterials, whose properties have been copiously investigated in recent years.1–3 The demand of lightweight and high strength materials, driven by automotive and aerospace industries, has motivated the effort of populating previously forbidden regions of the material property charts.4,5 In lattice material stiffness, strength and fracture response have been shown to depend upon geometry and nodal connectivity, with behaviors ranging from bending-dominated in foams to stretching-dominated in highly connected cellular solids such as octet-truss lattices.4,6–8 Recent advancements in fabrication capabilities have further spurred interest in fully exploiting architectured materials’ potential, exploring nanometer-scale lattices5,9–11 and hierarchical geometries.12,13 Structural metamaterials are perhaps even more appealing for their dynamic properties. Frequency-dependent forbidden elastic wave propagation and strongly directional behavior have been investigated in several lattice topologies and geometries,14–16 with possible applications in noise reduction, vibration control, and stress wave mitigation.2 Recently, enhanced functionality in lattices has been explored through nonlinearity to achieve amplitude-dependent response,17 tunable directivity with piezoelectric patches and shunted negative capacitance circuits,18 and large deformation effects.19 Another rather unexplored research direction in structural metamaterials is given by graded configurations, in which the periodic repetition of the same unit cell is replaced by a smooth grading of material properties or geometrical features. In this regard, gradient-index phononic crystals (GRIN PCs) can be designed to provide a refractive index profile able to focus elastic energy, realizing acoustic lenses,20 with recent promising extensions to piezoelectric energy harvesting.21,22 In structural lattices, grading the curvature of the beam elements has been numerically explored in undulated configurations.23 Although it was theoretically shown that structural metamaterials provide a disparate landscape of wave propagation properties, experimental validation is scarcely documented, with most of the tests performed to obtain transmissibility measures through a small number of sensors.24,25 When a full wavefield measurement is required, 3D scanning laser Doppler vibrometry (SLDV) is commonly used to measure the wave velocity field in predefined lattice locations.17,26 This approach presents a number of shortcomings, mainly related to the cost of the experimental apparatus and the challenge of focusing three laser beams onto the same measurement location, especially for small lattice beams. We recently proposed a different non-contact optical technique to achieve high-spatially resolved wavefield measurements, which is based on the optical measurement of the motion through high speed cameras and digital image correlation.27 We previously observed that non-periodic, graded configurations display enhanced filtering properties compared to the ones of their periodic counterparts.23 In this work, we discuss the experimental investigation of such filtering properties, obtained with the improved version of our optical, digital image correlation-based technique.

II. METHODS

The design of the graded structure relies on a numerical study of equivalent periodic undulated lattices, achieved by analyzing the bandgap locations of infinite periodic structures through a numerical implementation of Bloch’s analysis. For the experimental validation, we first use scanning
laser Doppler vibrometry (SLDV) to obtain transmissibility maps. Such maps illustrate the cumulative effect of the increasing undulation along the structure, informing the design of graded structures. Furthermore, a deeper understanding of the wave propagation phenomenon and the role of undulation gradation in filtering elastic waves is achieved with a second approach, based on the measurement of the in-plane wavefield in the lattices with a high speed camera. In this second experiment, we record the motion of the structure and assume that, at each recorded frame, such motion induces a small perturbation in the pixel intensities of the recorder digital images. Then, we correlate the digital images to indirectly infer conclusions on the structure’s motion. Figure 1 illustrates the structural lattices considered in this study. Undulated structures are obtained by imposing an initial curvature to the linking elements of a square, periodic straight reticulate. The undulation produces a periodic lattice if the imposed curvature varies periodically throughout the structure, while a graded, non-periodic structure is the result of a modulated curvature profile. Among all the possible undulated configurations, we focus our attention on the one inspired by the instability-induced pattern transformation in porous soft materials due to their interesting wave propagation properties. We start considering an infinite undulated periodic lattice, whose unit cell is represented in Fig. 2. The geometry of the unit cell is defined by $a$, which is twice the distance between two neighboring lattice intersections, $h$ the thickness of the lattice’s beams and $c$ the undulation amplitude.

In this work, we assume $a = 20.5$ mm and we target lattices with slender beams; therefore, we consider $h = 1$ mm. Also, intending to 3D print and test the lattice structures, we consider the lattice’s material to be ABSplus-P430 with tensile modulus $E = 2.2$ GPa and density $\rho = 1040$ kg/m$^3$. We perform a dispersion analysis of the structure by implementing a FE-based Bloch analysis and modeling the unit cell with Abaqus C3D6 6-node linear triangular prism elements. Wave attenuation due to material damping is expected in the 3D printed structures but not taken into account in the numerical model. However, we expect the material damping to induce the same attenuation in different tested structures, and thus, any wave filtering effect is only imputable to the structure’s undulated design. By sweeping the undulation amplitude $c$ from zero (straight lattice) to $c_{\text{max}} \approx 2$ mm, we construct the bandgap map of in-plane wave propagation in Fig. 2, obtained by identifying the width of the main bandgap for each value of the considered $c$. The main bandgap appears for $c > c_{\text{cr}}$, with $c_{\text{cr}}$ a certain critical value which in general depends on the slenderness of the beam and thus on the ratio $h/a$. The widest bandgap within the considered range of $c$ is obtained approximately for $c \approx 1.6$ mm, and it is about 5.5 kHz wide. In contrast, due the shape of the bandgap region in Fig. 2, bandgaps range from 15.7 kHz to 22.7 kHz, thus covering a wider 7 kHz frequency range. Therefore, one can speculate that a non-periodic structure with smooth graded undulation would benefit from the cumulative contributions of the local value of undulation, leading to an augmented elastic wave filtering capability compared to its periodic counterpart. Experimental validation of graded lattice performance is discussed in the second part of the present work, where tests are performed on two different lattices, a straight and a graded one.

We fabricate the lattices with a total of $N_U = 12$ unit cells each using a Stratasys Fortus 250mc 3D printer. The value $N_U$ is chosen in order to guarantee a smooth linear grading of the undulation in the range $c \in [0, 2]$ mm, given the manufacturing constraints in terms of the available printing area ($254 \times 254$ mm). During the fabrication process, the
lattices were lying flat on the 3D printer building surface, so as to minimize induced anisotropy due to uneven material deposition, which would bias the lattice’s in-plane dynamics. For the same reason, the highest degree of the fill-to-void ratio was imposed.

III. RESULTS AND DISCUSSION

A. Transmissibility maps

We use the experimental apparatus shown in Fig. 3 to measure the effect of the undulation gradation on the wave propagation attenuation. The lattice is hanging from a frame hold by thin cables to approximate free boundary conditions. The excitation is provided by a piezoelectric disk glued to the lattices’ edges. The piezoelectric disk generates a broadband signal, and then, the scanning head of the SLDV measures the transient response of the structure at different locations $x$ along its edge, where $x$ is a reference frame whose origin corresponds to the edge of the graded lattice with zero undulation. Due to the undulated edge’s surface, a retroreflective tape is applied to improve the laser’s signal quality. The response is recorded in the form of velocity, with a sampling rate of 256 kHz for 8 ms at each of 400 equally spaced locations from $x = 0$ to $x = L = 246$ mm. We define the transmissibility map $T(x, f)$, function of the frequency $f$ and the space variable $x$, as the ratio

$$T(x, f) = 20 \log_{10} \frac{|s(x, f)|}{|s(0, f)|},$$

where $s(x, f)$ is the Fourier transform of the signal recorded at the location $x$ and $s(0, f)$ is the Fourier transform of the reference signal recorded at $x = 0$. A comparison between the transmissibility maps of the straight and graded lattices is shown in Fig. 4. The maps show that the graded lattice achieves a dramatic drop of transmissibility (between ~40 dB and ~60 dB) in the range between 20 kHz and 27.5 kHz at $x = L$, thus providing a 7.5 kHz wide wave attenuation range. Moreover, such a drop in transmissibility is particularly visible at 20 kHz for $x > 150$ mm, the frequency at which the graded lattice is most effective in filtering elastic waves. In comparing the experimental and theoretical results, we remark that the analysis of the bandgap map predicts wave attenuation in a 7 kHz wide range of frequencies, which is in excellent agreement with experimental validation. On the other hand, such a range is shifted upwards in frequency, which might be explained partly by the uncertainty in the material properties of the 3D printed material, especially the effective elastic modulus.

B. DIC-based wavefield measurement

Based on the information given by the transmissibility map for the graded lattice, we design a second experiment to measure the in-plane wavefield. We target the response of the system at 20 kHz, frequency at which the drop of transmissibility is the largest. The experimental setup, shown in Fig. 5, employs a single high-speed camera (Photron

![Fig. 3. Experimental setup for the measurement of transmissibility maps $T(x, f)$.](image)

![Fig. 4. Transmissibility maps $T(x, f)$ for the straight (a) and graded (b) lattices. The graded configuration guarantees a wide transmissibility drop from 20 to 27.5 kHz. The difference in transmissibility between the straight (solid red line) and graded (dotted blue line) lattices is shown in detail in (c) for $x = 23.5$ mm.](image)
Fastcam SA1.1) to record the motion of the structure. Adequate light is provided by two high intensity lights (Lowel Pro-light). Elastic waves are excited by actuating an ultrasonic piezo-transducer (APC 90-4060) tuned to resonate at $f = 20$ kHz. In order to improve the coupling between the structure and the actuator, we apply a 2.5 kg preload by using weights sitting on top of a horizontal bar, which is free to slide vertically. A laser Doppler vibrometer (LDV) monitors the lateral displacement at a point of the lattice’s edge.

A switch initializes the camera’s recording, and then, the camera triggers the acquisition unit (DAQ), which generates the excitation signal. This signal is first amplified and then sent to a piezoelectric actuator (PZT) placed at the bottom edge of the lattice. The lattice is preloaded to ensure firm contact with the actuator by means of weights sitting on a horizontal bar, which is free to slide vertically. A laser Doppler vibrometer (LDV) monitors the lateral displacement at a point of the lattice’s edge.

Concurrently, the DAQ records the voltage output from a laser Doppler vibrometer (LDV, Polytec PDV 100), which monitors the structure’s response at one point on the lattice’s side. In choosing the size of the measurement area, we have to consider the high-speed camera’s limitations in reading and storing the information while recording. Higher sampling rates force us to reduce the frame size and thus the number of the recorded pixels within the same image, while choosing larger frame sizes implies reducing the frame rate. This inverse relationship between the frame rate and frame size would prevent us from measuring large enough wavefields with a sufficient temporal resolution. Nevertheless, such limitations are overcome by effectively increasing both spatial sampling and temporal sampling, as discussed in our previous works.27,31

The measurement domain is divided into 23 tiles, each corresponding to an array of $2 \times 14$ lattice intersections. For each tile, the experiment is performed separately, and then, the data are stitched together to retrieve the full wavefield. We choose to track 20 evenly spaced points between each intersection, which sums up to roughly 8600 measurement points for the total considered measurement area. The experiments are performed at a sampling rate of $f_s = 75$ kHz. An effective sampling rate $f_{s,\text{eff}} = 2f_s = 150$ kHz is realized by properly interleaving two different sets of measurements, which differ by a certain delay $t_d = 1/(2f_s)$ between the beginning of the camera measurement and the onset of the excitation. The delay time $t_d$ is imposed by conveniently programming the DAQ. For each set of measurement, we consider 5 averages which help improving the signal-to-noise ratio by reducing the uncorrelated noise. We target the behavior of the system to a narrowband excitation, and thus, we excite the system with a 11-cycle tone-burst at 20 kHz. In order to obtain a sufficiently large excitation signal, the ultrasonic piezo-transducer is coupled to a resonator to amplify its response. The piezo-resonator assembly is tuned to have its first resonant frequency at 20 kHz by properly selecting the resonator’s length and conveniently preloading the assembly. The raw data, in the form of pixel intensity variations over different frames, are collected and pre-processed by correlating the frame set, averaging and interleafing the two measurement sets.27 The pre-processed data are then post-processed by filtering the data in the frequency domain around the excitation frequency to improve the signal-to-noise ratio. Owing to the highly discontinuous nature of the lattice structure, the result visualization is considerably improved by interpolating the data onto a rectangular $121 \times 461$ grid of points, as shown in Fig. 6(a). Finally, a
moving average filter is applied to the interpolated data to produce the plots shown in Fig. 6(b), which shows the wavefields in both straight and graded configurations, together with the original measurement points at different time instants. We can successfully track the wavefront from the excitation location to the opposite side of the structure. We also remark on the strong directional nature of the wavefront, as expected for straight lattices. The wavefield of the graded lattice, on the other hand, starts differing quite remarkably from the one in the straight lattice already few intersections away from the excitation location, as the effect of the undulation gets strong enough. This effect becomes remarkable halfway through the lattice, preventing the energy carried by the elastic waves from propagating any further, effectively confining it to the first half of the structure.

IV. CONCLUSION

In conclusion, we experimentally validated the in-plane filtering properties of undulated lattices, showing that single-phase structural metamaterials can be conveniently designed in graded configurations by slowly varying the curvature of the lattice elements along one direction. We showed that the design of the graded lattice is informed by the dispersion analysis of the infinite periodic undulated lattice, with a thorough inspection of the bandgap map representing the relationship between the bandgap width and the beam element’s curvature. Then, we compute transmissibility maps by measuring the velocity field at one edge of a graded and an equivalent straight lattice with an SLDV system, identifying a frequency range with transmission attenuation in the graded configuration. Finally, we employ a high speed camera and digital image correlation to measure the full wavefield for a narrow-band excitation with a frequency spectrum falling within the large attenuation frequency range in both lattices, tracking how in-plane elastic waves are attenuated in the graded one only due to the gradual change in geometry. Future research directions include extending the full wavefield measurement herein discussed for the study of the directional properties of graded lattices, with applications in superior energy guiding, energy focusing, and energy harvesting.

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