AE 2020 Quiz II
November 1, 2000
Closed Book; Two pages of notes and calculator allowed.
Please do not ask GRA for help or clarification.

1) Describe Kutta condition.

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2) Describe how lift is generated by an airfoil impulsively starting from rest.

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3) Consider the stream function: $y = \left(r - \frac{1}{r}\right)\sin q$. Given $v_r = \frac{1}{r}\frac{\partial y}{\partial q}$ and $v_q = -\frac{\partial y}{\partial r}$

a) Determine if this flow field satisfies the continuity equation:

$$\vec{\nabla} \cdot \vec{V} = \frac{1}{r}\left[\frac{\partial (r v_r)}{\partial r} + \frac{\partial v_q}{\partial q}\right] = 0$$

b) Determine if this flow field satisfies irrotationality: $\left(\frac{\partial (r v_q)}{\partial r} - \frac{\partial v_r}{\partial q}\right) = 0$

c) Determine if this flow field both satisfy the Laplace's equation:

$$\frac{\partial^2 y}{\partial r^2} + \frac{1}{r}\frac{\partial y}{\partial r} + \frac{1}{r^2}\frac{\partial^2 y}{\partial q^2} = 0$$

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3) An airfoil has the following mean camber line shape: $Z_c(x) = 4hx(1-x)$

Here h is a constant. The distances Z_c and x have been non-dimensionalized by the chord, so that c = 1, and $1 \geq x \geq 0$.

a) Show that the lift coefficient for this airfoil is given by $C_l = 2\pi\alpha + 4\pi h$
b) Sketch the Z_c vs. x curve and C_l vs. α curve for positive and negative values of h.
c) Determine the moment coefficient at the airfoil leading edge.
d) Determine the center of pressure in terms of α and h.

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Some relations you may need:

$$\int_0^p \cos^2 q \, dq = \frac{p}{2}$$

$$\int_0^p \cos q \cos 2q \, dq = 0$$

$$A_0 = a - \frac{1}{p}\int_0^p \frac{dZ_c}{dx} dq$$

$$A_n = \frac{2}{p}\int_0^p \frac{dZ_c}{dx} \cos nq \, dq \qquad n = 1,2...$$

$$\frac{x}{c} = \frac{1-\cos q}{2}$$

$$C_l = 2pA_0 + pA_1$$

$$C_{m,L.E.} = -\frac{C_l}{4} - \frac{p}{4}(A_1 - A_2)$$