Newtonian Flow Theory

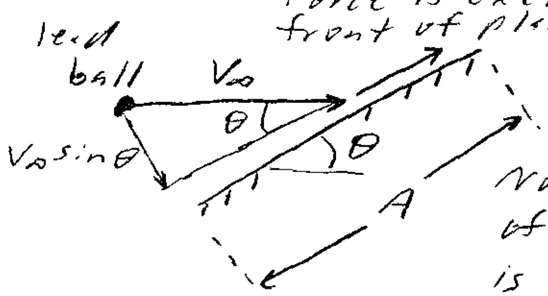
- For conceptual and preliminary design purposes it is normally impractical to use non-linear CFD methods for a large number of candidate geometries.

- Several simple and rapid methods exist for <u>surface pressure</u> prediction in hypersonic flow. The first of these "surface inclination" methods is <u>Newtonian Flow</u> theory.

* In 1687, Newton developed a fluid mechanics theory for <u>low-speed</u> aerodynamics. The theory is not accurate... except, ironically for hypersonic flow!

- Newton's Theory states that an object hitting a surface would lose all of its momentum normal to the surface but keep all of its tangential momentum.

• Consider a lead ball (no bounce), hitting an inclined plane:



No objects hit the back of the plane. This side is in the "shadow" of the front surface.

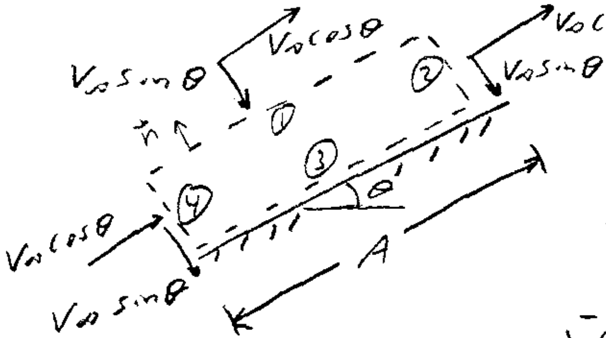
Force on back side = 0

- Force exerted by the lead ball (analogous to fluid particle) is computed by conservation of momentum.

- Tangential component ($\cos\theta$) is not affected since its conserved.

• Consider the particles to be entering a control volume with area A:

normal to inclined plane



Rate of change of normal momentum through the control volume.

Normal mom. at wall

$$\underbrace{-\rho_\infty V_\infty \sin\theta\, A}_{\substack{\text{Mass Flux} \\ \iint(\vec{V}\cdot\hat{n})\,dA}}\ \underbrace{V_\infty \sin\theta - \underbrace{0}_{③}}_{\substack{\text{Velocity goes} \\ \text{from this value} \\ \text{to zero}}}$$

• Normal momentum flux through ② and ④ cancel each other.

$$= -\rho_\infty V_\infty^2 \sin^2\theta\, A$$

• Newton's 2nd Law

$$F = \text{Time rate of change of momentum}$$

$$F = -\rho_\infty V_\infty^2 \sin^2\theta\, A = \underbrace{(P_\infty - P)A}$$

Net pressure force on the control volume (= force on surface)

P = Pressure on surface.

• Convert to pressure coefficient:

$$C_p \equiv \frac{P - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\rho_\infty V_\infty^2 \sin^2\theta}{\frac{1}{2}\rho_\infty V_\infty^2}$$

$$\boxed{C_p = 2\sin^2\theta} \leftarrow \text{Newton's sin-squared law}$$

(Newtonian Flow C_p expression)

Q: Why isn't this exact for inviscid flow? What happens in the general case of M_∞ not $\gg 1$?

* When applying this result to hypersonics, note:
  - θ is the angle between the freestream direction and the surface angle.
    (Not the same θ as in oblique shock equations.)
  - θ is local inclination angle so Newtonian theory is a local surface inclination method. C_p only depends on θ.
  - C_p is not a function of Mach number
  - We extend the result to 3-D & the normal velocity is $\vec{V}\cdot\vec{n}$ instead of $V_\infty \sin\theta$

For 3-D, Newtonian Flow: $\boxed{C_p = 2\frac{|\vec{V}_\infty \cdot \vec{n}|^2}{V_\infty^2}}$

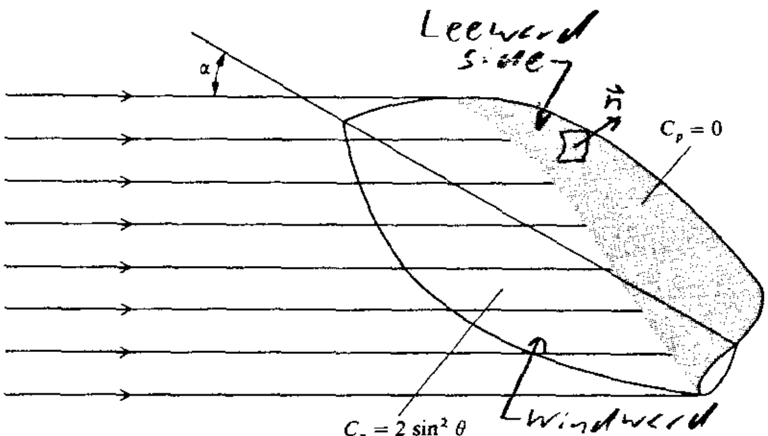


FIGURE 3.4
Shadow region on the leeward side of a body, from newtonian theory.

- In the shadow region, $\vec{V}_\infty \cdot \vec{n} > 0$
  No flow strikes the body so we can approximate, $P \approx P_\infty$ and $C_p = 0$ in shadow region

If $M_\infty \gg 1$, P on leeward side << P on windward side

Roughly: $P_{leeward} \lesssim P_\infty$

Normally both $P_{leeward}$ and $P_\infty << \frac{1}{2}\rho_\infty V_\infty^2$

* How accurate is Newtonian Theory?

- For a flat plate works very well. For most other bodies accuracy is poor if $M_\infty \lesssim 5$
- Accuracy improves as M_∞ increases

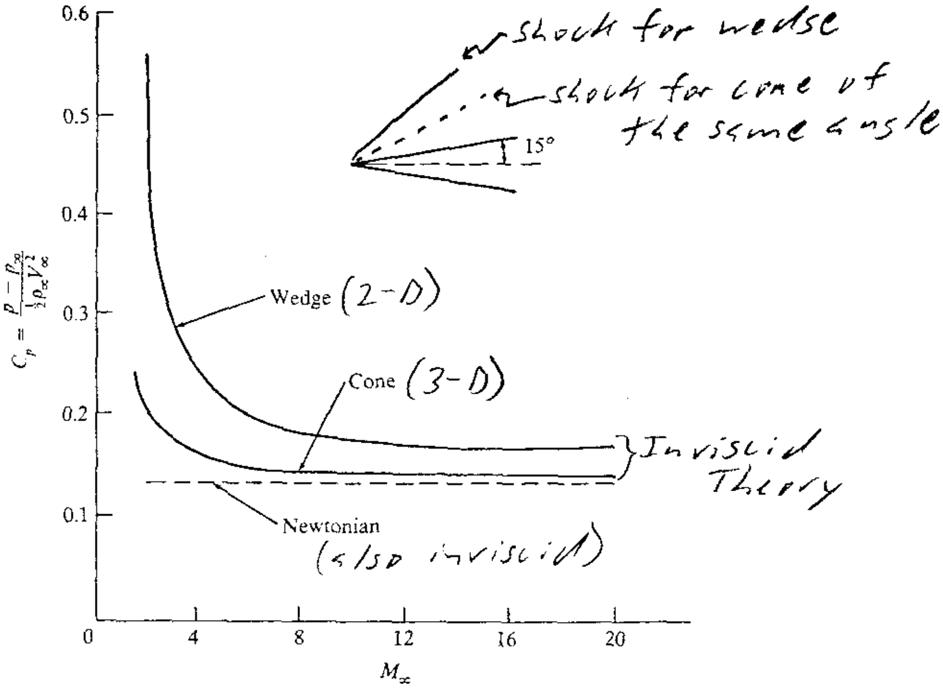
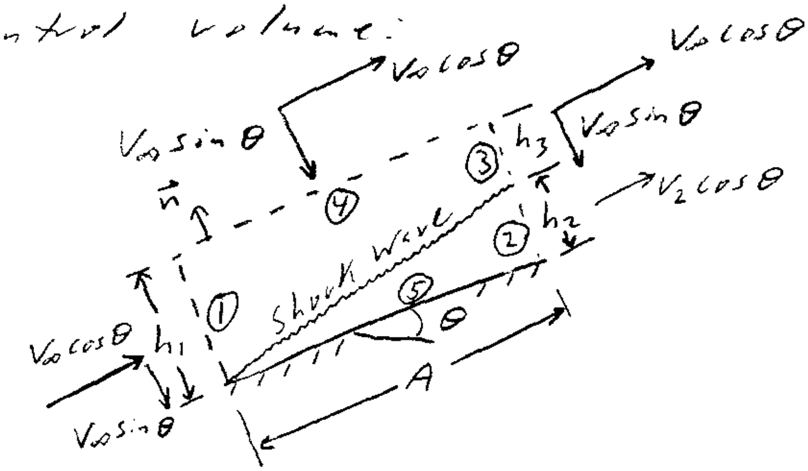


FIGURE 15.10
Comparison between newtonian and exact results for the pressure coefficient on a sharp wedge and a sharp cone.

- Tends to underpredict C_p (and thus C_L and C_D) for sharp bodies
- Tends to overpredict C_p (and thus C_L and C_D) for blunt bodies

Q: Why is Newtonian Theory only valid in hypersonic flow:

* In the real case, for $M_\infty > 1$ a shock wave will penetrate at least one face of the control volume:



Rate of change of normal momentum on all sides of the control volume.

$$-\underbrace{(\rho_\infty V_\infty \sin\theta\, A)(V_\infty \sin\theta)}_{④} + \underbrace{(\rho_\infty V_\infty \cos\theta\, h_3) V_\infty \sin\theta}_{③}$$

$$+ \underbrace{(\rho_2 V_2 \cos\theta\, h_2)(0)}_{②} - \underbrace{(\rho_\infty V_\infty \cos\theta\, h_1) V_\infty \sin\theta}_{①} + \underbrace{0}_{⑤}$$

$$= -\rho_\infty V_\infty^2 \sin^2\theta\, A - \underline{\underline{\rho_\infty V_\infty^2 \cos\theta \sin\theta\,(h_1 - h_3)}}$$

Term not considered in Newtonian Theory

Thus, • Newtonian assumptions imply that $h_1 = h_3$ (i.e. that shock wave lays directly on the surface)

• Newtonian Theory (falsely) assumes that tangential momentum is conserved. In real (subsonic, supersonic, and hypersonic) flows, tangential momentum changes & is balanced by changes in pressure on the control volume.

Modified Newtonian Theory

- Once the value of Newtonian Theory was realized in the 1950's, improvements to the theory were developed.
- For a blunt body, Modified Newtonian Theory is more accurate than Newtonian Flow.
- The C_p expression is no longer independent of M_∞.

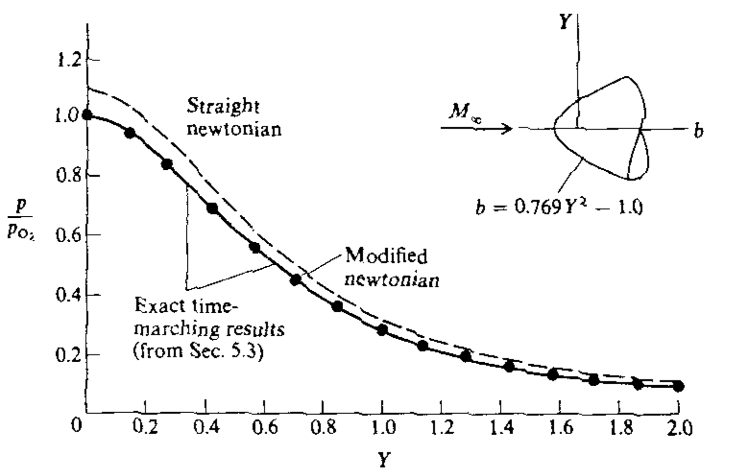


FIGURE 3.8
Surface pressure distribution over a paraboloid at $M_\infty = 8.0$; p_{O_2} is the total pressure behind a normal shock wave at $M_\infty = 8.0$.

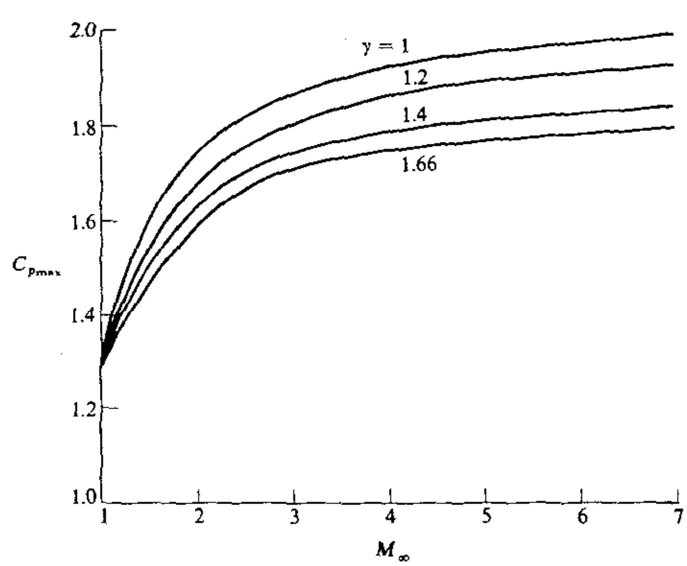
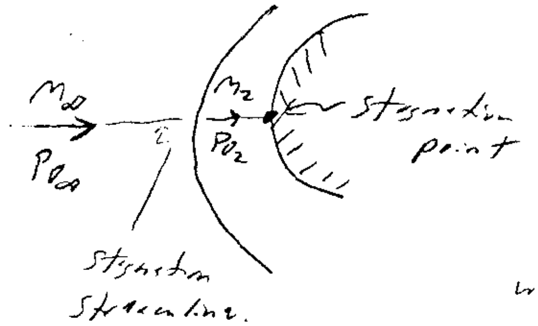


FIGURE 3.7
Variation of stagnation pressure coefficient with M_∞ and γ.

- Consider a blunt body in hypersonic flow.



- The fluid elements along the stagnation streamline experience a normal shock which drops the stagnation pressure to P_{0_2}.

- Recall that in general $C_p \equiv \dfrac{P - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \dfrac{2}{\gamma M_\infty^2}\left(\dfrac{P}{P_\infty} - 1\right)$

- Thus at the stagnation point (where $P = P_{0_2}$) the pressure coefficient is:

$$C_{p_{max}} = \frac{2}{\gamma M_\infty^2}\left(\frac{P_{0_2}}{P_\infty} - 1\right)$$

where P_{0_2} is the stagnation pressure after the freestream flow undergoes a normal shock.

- P_{0_2} is a function of M_∞:

$$\frac{P_{0_2}}{P_\infty} = \left[\frac{(\gamma+1)^2 M_\infty^2}{4\gamma M_\infty^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1-\gamma+2\gamma M_\infty^2}{\gamma+1}\right]$$ ← Assumes γ = const. May be a poor assumption

- However, straight Newtonian theory says that at the stagnation point (i.e. $\theta = 90°$),

$$C_{p_{stag}} = 2 \quad \text{regardless of } M_\infty$$

- Lester Lees (1955) proposed Modified Newtonian Theory which provides the correct C_p at the stag. point & improved C_p's around blunted bodies:

Modified Newtonian $\Rightarrow$ $\boxed{C_p = C_{p_{max}} \sin^2\theta}$ ← Find $C_{p_{max}}$ from $\frac{P_{0_2}}{P_\infty}$ eqn. Or predict high temp. effects to set $C_{p_{max}}$

Note that when we combine the $\frac{P_{02}}{P_\infty}$ expression with $C_{p_{max}} = \frac{2}{\gamma M_\infty^2}\left(\frac{P_{02}}{P_\infty} - 1\right)$, we find that

when $M_\infty \to \infty$: $C_{p_{max}\,M_\infty\to\infty} = \left[\frac{(\gamma+1)^2}{4\gamma}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{4}{\gamma+1}\right]$

1) In this case $C_{p_{max}}$ and $C_p = C_{p_{max}} \sin^2\theta$ are no longer dependent on Mach number

2) For $\gamma = 1.4$, $C_{p_{max}} \to 1.839$ as $M_\infty \to \infty$

3) <u>As $\gamma \to 1$ and $M_\infty \to \infty$</u> we get $C_{p_{max}} \to 2$

and $\boxed{C_p = 2\sin^2\theta}$

Newtonian Flow Result