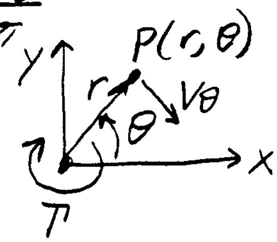
Thin Airfoil Theory - Introduction

- We have now seen how sources, sinks, doublets and/or vortices can be superimposed to describe incompressible flows

- Classical Thin Airfoil Theory uses vortices to describe flows around thin ($\frac{t}{c} \lesssim 10\%$) airfoils.

- Rather than considering the <u>point vortices</u> described earlier:

Recall: $\varphi = \frac{\Gamma}{2\pi}\ln(r)$ and $\psi = -\frac{\Gamma\theta}{2\pi}$

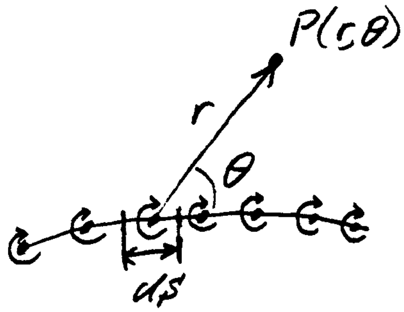


$$\underbrace{V_r = 0 \qquad V_\theta = -\frac{\Gamma}{2\pi r}}_{\text{Velocity induced at P by vortex}}$$

where $\Gamma \equiv$ Point vortex strength

($\Gamma > 0$ for clockwise circulation)

• We instead consider a <u>vortex sheet</u>:



- Strength of <u>each</u> vortex is vanishly small. We instead describe:

<u>$\gamma = \gamma(s) \equiv$ Strength of Vortex Sheet per unit span</u>

Note: $\gamma(s)$ may vary along the vortex sheet.

- Consider the segment of length ds shown above. The strength of that segment is then $\underbrace{\gamma\, ds}_{\sim \Gamma \text{ for that part of the vortex sheet.}}$

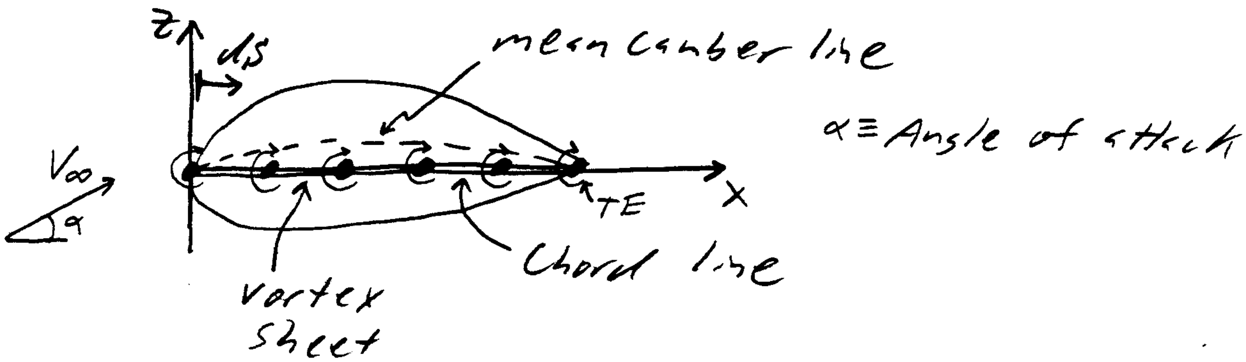
- Thus, replacing Γ with $\gamma\, ds$ in the point vortex expressions earlier gives:

$$\underbrace{V_r = 0 \qquad V_\theta = -\frac{\gamma\, ds}{2\pi r}}_{\text{Velocity induced at point } P \text{ by length } ds \text{ of the vortex sheet.}}$$

- Note: All other parts of the vortex sheet also influence the flow at point P.

Thin Airfoil Theory derivation Strategy:

1) Place a vortex sheet on the chord line to model the circulation generated by the airfoil



2) We solve for the variation of $\gamma(s)$ along the vortex sheet such that
   - The mean camber line becomes a streamline of the flow
   - The Kutta Condition is satisfied. We will see that this means $\gamma(TE)=0$

3) Use the $\gamma(s)$ we have solved for to get the velocity distribution along the airfoil.

4) Use Bernoulli to find P as a function of velocity.

5) Integrate pressures to find C_ℓ and C_m

C_m: sectional pitching moment coeff.

C_ℓ: sectional lift coefficient