(1) $c = 4\,ft$ $\qquad P_\infty = 2000\ lbf/ft^2$

$V_\infty = 150\ MPH = 220\ ft/sec$ $\qquad \rho_\infty = 0.002311\ slugs/ft^3$

$\alpha = 8° = 0.13963$ radians $\qquad$ Symmetric airfoil

a) $c_\ell = 2\pi\alpha \Rightarrow \boxed{c_\ell = 0.87730}$

$$c_\ell \equiv \frac{L'}{\frac{1}{2}\rho_\infty V_\infty^2 c} \Rightarrow L' = c_\ell\left(\tfrac{1}{2}\rho_\infty V_\infty^2 c\right) \Rightarrow \boxed{L' = 196.26\ lbf/ft}$$

b) $$L' = \rho_\infty V_\infty \Gamma \Rightarrow \Gamma = \frac{L'}{\rho_\infty V_\infty} \Rightarrow \boxed{\Gamma = 386.01\ \frac{ft^2}{sec}}$$

c) $$\underbrace{c_{m_{LE}} = -\frac{c_\ell}{4}}_{\text{Symmetric airfoil}} \Rightarrow \boxed{c_{m_{LE}} = -0.2193}$$

d) $\boxed{c_{m_{c/4}} = 0}$ for symmetric airfoil

e) For a symmetric, thin airfoil

$$\gamma(\theta) = 2\alpha V_\infty \frac{(1+\cos\theta)}{\sin\theta}$$

$$\xi = \frac{c}{2}(1-\cos\theta)$$

At mid chord $\xi = \frac{c}{2}$, $\theta = \frac{\pi}{2}$

$$\gamma\left(\tfrac{\pi}{2}\right) = (2)(0.13963)(220)\frac{1+0}{1} \Rightarrow \boxed{\gamma\left(\tfrac{\pi}{2}\right) = 61.436\ \frac{ft}{sec}}$$

②

$$z = k\sin^2\theta$$

$$\frac{dz}{dx} = \frac{dz}{d\theta}\frac{d\theta}{dx} \qquad \frac{dz}{d\theta} = 2k\sin\theta\cos\theta$$

$$x = \frac{c}{2}(1-\cos\theta) \Rightarrow dx = \frac{c}{2}\sin\theta\, d\theta \qquad \frac{d\theta}{dx} = \frac{2}{c\sin\theta}$$

$$\frac{dz}{dx} = 2k\sin\theta\cos\theta \times \frac{2}{c\sin\theta} = \frac{4k}{c}\cos\theta$$

$$C_{m_{ac}} = C_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1)$$

$$A_n = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos n\theta\, d\theta = \frac{8k}{\pi c}\int_0^{\pi}\cos\theta\cos n\theta\, d\theta$$

if $n=1$ $\int \rightarrow \pi/2$ $\qquad A_1 = \frac{4k}{c}$

if $n=2$ $\int \rightarrow 0$ $\qquad A_2 = 0$ $\qquad \boxed{C_{m_{ac}} = -\frac{\pi k}{c}}$

② (continued)

Let's now find

$$\alpha_{L0} = -\frac{1}{\pi}\int_{\theta=0}^{\pi} \frac{dz}{dx}(\cos\theta - 1)\,d\theta$$

$$\alpha_{L0} = -\frac{4K}{\pi c}\int_{\theta=0}^{\pi} (\cos^2\theta - \cos\theta)\,d\theta$$

$$\alpha_{L0} = \left(\frac{4K}{\pi c}\cancel{\sin\theta}\right)\Bigg|_{\theta=0}^{\pi} \overset{0}{} - \left(\frac{4K}{\pi c}\right)\left(\frac{1}{2}\right)\int_{\theta=0}^{\pi}(1 + \cos 2\theta)\,d\theta$$

$$\alpha_{L0} = -\frac{2K}{\pi c}\left[\theta + \frac{1}{2}\cancel{\sin 2\theta}\right]_{\theta=0}^{\pi} \Rightarrow \underline{\alpha_{L0} = -\frac{2K}{c}}$$

$$\underline{c_l} = 2\pi(\alpha - \alpha_{L0}) = \underline{2\pi\left(\alpha + \frac{2K}{c}\right)}$$

$$x_{cp} = \frac{c}{4}\left[1 + \frac{\pi}{c_l}(A_1 - A_2)\right]$$

Now substitute in everything

$$x_{cp} = \frac{c}{4}\left[1 + \frac{1}{2\left(\alpha + \frac{2K}{c}\right)}\left(\frac{4K}{c}\right)\right]$$

$$\boxed{x_{cp} = \frac{c}{4}\left[1 + \frac{2K}{c\left(\alpha + \frac{2K}{c}\right)}\right]}$$

(3) For the airfoil given the slope of the mean camber line is

$$\begin{cases} \dfrac{dz}{dx} = \dfrac{.05c}{c/4} = 0.2 & \text{for } 0 \le x < \frac{c}{4} \quad \text{or} \quad 0 \le \theta < \frac{\pi}{3} \\ \dfrac{dz}{dx} = 0 & \text{for } \frac{c}{4} \le x < c/2 \quad \frac{\pi}{3} \le \theta < \frac{\pi}{2} \\ \dfrac{dz}{dx} = \dfrac{-.05c}{c/2} = -0.1 & \text{for } \frac{c}{2} \le x \le c \quad \frac{\pi}{2} \le \theta \le \pi \end{cases}$$

Note that $x = \frac{c}{2}(1 - \cos\theta)$

$$\theta = \arccos\left[1 - \frac{2x}{c}\right]$$

From eqn 4.61 in text.

$$\alpha_{L0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta - 1)\,d\theta$$

$$\alpha_{L0} = -\frac{1}{\pi}\int_0^{\pi/3} \text{---} \; - \frac{1}{\pi}\int_{\pi/3}^{\pi/2} \text{---} \; - \frac{1}{\pi}\int_{\pi/2}^{\pi} \text{---}$$

For each of these integrals $\frac{dz}{dx}$ is constant so

$$\alpha_{L0} = -\frac{1}{\pi}(0.2)\left[\sin\theta - \theta\right]\Big|_0^{\pi/3} - \frac{1}{\pi}(0) - \frac{1}{\pi}(-0.1)\left[\sin\theta - \theta\right]\Big|_{\pi/2}^{\pi}$$

$$\alpha_{L0} = 1.1534\times10^{-2} - 8.1831\times10^{-2}$$

$$\underline{\alpha_{L0} = -0.070297 \text{ radians}} = 4.03°$$

$$c_l = 2\pi(\alpha - \alpha_{L0}) = 2\pi(0.10472 + .070297)$$

$$\boxed{c_l = 1.1}$$

b) For a thin symmetric airfoil $c_l = 2\pi\alpha = 2\pi(0.10472)$

$$\boxed{c_l = 0.658}$$