Courses

April 14, 2008

Select from the following course list.

- **AE-3125:** *Aerospace Structural Analysis*, sec. 3.
- **AE-6107:** *Analysis of Aerospace Structural Elements*, sec. 4.
- **AE-6210:** *Advanced Dynamics*, sec. 5.

1 Mandatory Homework Format

The homework you turn in are a good measure of the quality of your work and the effort you put into a course. Make every effort to present your work in the best possible manner.

1. Use engineering paper only. **PRINT** your name clearly on the **TOP RIGHT-HAND** corner of each and every page. Work only on one side of the sheets. STAPLE all sheets together.

2. Be careful about neatness and being organized. Please use *pencil and clean erasure*, **ink is not acceptable**.

3. **Neatness counts**, it is a sound approach to engineering.

4. Clearly indicate your final results by putting a **box around your answers**.

5. Do not forget to keep track of **UNITS** during your calculations.
   (a) It is a convenient way to *uncover math errors.*
   (b) Please indicate units for your final answers: **a final answer without units is not an answer.**

6. **Describe your solution process first:**
   (a) Provide a *sketch of the problem* with all relevant information.
   (b) If applicable, **draw a free body diagram.**
   (c) Derive the *equations of the problem.*
   (d) Discuss your *approach to solving* them (analytical, numerical, etc).
   (e) Provide expressions for all the results you are plotting.
   (f) Comment on the **physical nature and significance of your answers.**
   (g) Please do not give an answer that makes no sense! It clearly indicates you do not understand the material.

7. A plot conveys abstract information in a graphical manner. A **plot must be drawn with the help of a software package** (Matlab, Maple, etc.). Freehand drawing is a sketch, not a plot. Consider the plot shown in fig. 1.
   (a) Both x and y axes must be **properly labeled**; provide **UNITS**. If it is a non-dimensional quantity your are plotting, say so.
   (b) If more than one curve appears on the plot, make sure to **clearly differentiate them**. Use *different line styles or symbols.*
   (c) Provide a **caption** that explains what quantities are plotted along the x and y axes. If more than one curve appears on the plot, provide a **legend** to explain the *meaning of each curve.*

8. If you are using a software package (Matlab, Maple, etc.) as a part of your solution process, **include the input/output files to these packages as part of your submission**.

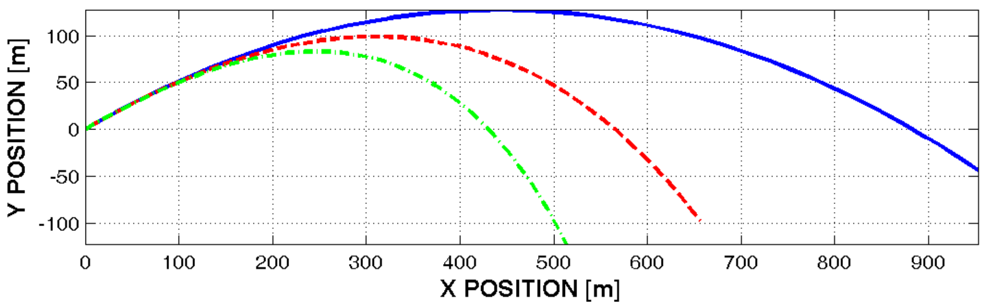

Figure 1: Trajectory of the particle: ignoring air friction (solid line); with friction coefficient k = 0.001 kg/m (dashed line); and k = 0.002 kg/m (dashed-dotted line).

2 Statement of Academic Honesty

The honor system is assumed.

1. You may, and are encouraged to discuss how to work out the problem sets with your classmates. Classmates are an excellent source of information. *There is an obvious difference between a constructive discussion about a homework with a friend, and copying your friend's homework.*

2. Of course, **copying is not permitted**.

3. For more information see the Academic Honor Code.

3 AE-3125: Aerospace Structural Analysis

Here is where to find more information about

- The mandatory homework format, see section 1.
- The statement of academic honesty, see section 2.
- Lectures: M-W 10:00 to 10:55, F 9:05 to 10:55, Guggenheim Building, Room 244.
- Teaching assistant: . Office hours: .

3.1 Topics to be covered in this course

1. Review of the basic equations of elasticity.
2. Review of classical Euler-Bernoulli beam theory.
3. Euler-Bernoulli beam theory for beams with unsymmetrical cross-sections..
4. Torsion of thin walled beams.
5. Shear flows due to transverse shearing forces in thin-walled beams of open and closed cross-sections.
6. Coupled bending-torsion problems.
7. Energy and variational principles.
8. Analysis of wing structures under aerodynamic loading.

3.2 Grading Policy

The overall numerical grade for this course will be computed using the weighting factors shown in table 1.

8-10 Homework Assignments	30%
4 Quizzes	40%
1 Final exam	30%
TOTAL	**100%**

Table 1: Grade weighting factors

3.3 Homework

Homework will be assigned on a weekly basis. Homework is assigned on Friday lectures and is due the next Friday, in class. Homework is a vital part of the learning process, **and 30% of your final grade.** To make sure no homework is *forgotten*, an aging factor will be built into the grading as shown in table 2. Do not forget to use the mandatory homework format, see section 1.

Date homework is turned in	**Aging factor**
On the due date	actual grade
Up to the Monday after due date	actual grade - 1/10
Up to the Wednesday after due date	actual grade - 2/10
Never turned in	0/10

Table 2: Aging factor for homework

3.4 Quizzes and Final

Four one-hour quizzes will be held at the dates posted on the schedule below. Quizzes are closed book and closed notes, but open minds. Three crib sheets, hand written front and back, are allowed at each quiz. The comprehensive final exam will be held on Apr. 30^{th} 14:50 - 17:40. The format of the final is identical to that of the quizzes, but five crib sheets are allowed.

3.5 Reference books

The following reference text books are a good source of information for the class: [1, 2, 3, 4, 5, 6].

3.6 Course Schedule

Week	Dates	Reading Assignments	Homework
1	Jan. 7^{th} - 11^{th}	Chapter 1	
2	Jan. 14^{th} - 18^{th}	Chapter 1	Hw # 1, see sec. 3.6.1
3	Jan. 21^{st} - 25^{th}	Chapter 2	**Quiz # 1:** Jan. 25^{th}
4	Jan. 28^{th} - Feb. 1^{st}	Chapter 4	Hw # 2, see sec. 3.6.2
5	Feb. 4^{th} - 8^{th}	Chapter 5	Hw # 3, see sec. 3.6.3
6	Feb. 11^{th} - 15^{th}	Chapter 6	Hw # 4, see sec. 3.6.4
7	Feb. 18^{th} - 22^{nd}	Chapter 6	Hw # 5, see sec. 3.6.5
8	Feb. 25^{th} - 29^{th}	Chapter 7	Hw # 6, see sec. 3.6.6
9	Mar. 3^{rd} - 7^{th}	Chapter 7	**Quiz # 3:** Mar. 7^{th}
10	Mar. 10^{th} - 14^{th}	Chapter 8	
11	Mar. 17^{th} - 21^{st}		Enjoy the **Spring Break**
12	Mar. 24^{th} - 28^{th}	Chapter 8	Hw # 7, see sec. 3.6.7
13	Mar. 31^{st} - Apr. 4^{th}	Chapter 8	
14	Apr. 7^{th} - 11^{th}	Chapter 11	**Quiz # 4:** Apr. 11^{th}
15	Apr. 14^{th} - 18^{th}	Chapter 11	Hw # 8, see sec. 3.6.8
16	Apr 21^{st} - 25^{th}		**Final**: Apr. 30^{th} 14:50 - 17:40

3.6.1 Homework # 1

Solve the following problems in the class notes: 1.3.3, 1.3.7 and the problems below. **Due date:** Jan. 23^{rd} 2008.

Given a state of stress defined by: σ_1= 350 MPa, σ_2=- 600 MPa, $\sigma_3 = 50$ MPa, $\tau_{12} = -45$ MPa, $\tau_{13} = 100$ MPa and $\tau_{23} = 100$ MPa, *(1)* Determine the principal stresses, *(2)* Determine the principal stress directions. HINT: you should consider using a software package to handle the computations.

Consider the T-V rosette shown in fig. 1.22 D. The measured data is $e_1 = 1200\mu$, $e_2 = 900\mu$, $e_3 = -240\mu$ and $e_4 = 800\mu$. Use a least square approach to solve this problem. *(1)* Find the state of strain at this location. *(2)* Draw Mohr's circle for this state of strain. *(3)* Find the orientation of the principal strain directions, and *(4)* the principal strains.

3.6.2 Homework # 2

Solve the following problems in the class notes: 2.1.4, 2.1.5, 4.3.5, 4.3.9. **Due date:** Feb. 8^{th} 2008.

3.6.3 Homework # 3

Solve the following problems in the class notes: 5.4.1, 5.5.10, and the problem below. **Due date:** Feb. 15^{th} 2008.

The cantilevered beam depicted in fig. 2 is of length L, uniform bending stiffness H_{33}^c, and is subjected to a uniform distributed load p_0. A concentrated spring of stiffness constant k is connected to the beam at a distance a from its root. *(1)* Find the exact solution of the

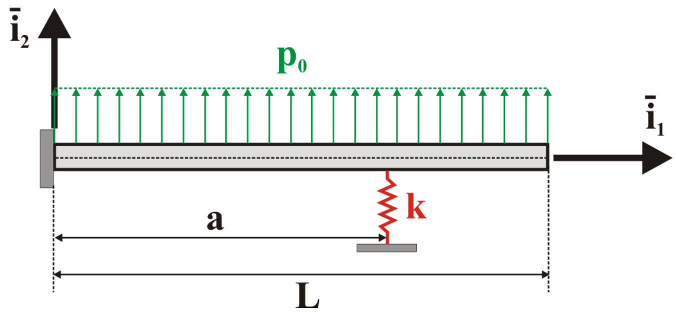

Figure 2: Cantilevered beam with concentrated spring.

problem. It will be convenient to define the non-dimensional spring constant $k^* = ka^3/H^c_{33}$. *(2)* Plot the distribution of transverse displacement for the beams. Use $L/a = 3$ and $k^* = 100$. *(3)* Plot the distribution of bending moment for the beams. *(4)* Plot the distribution of shear force for the beams. *(5)* Find the value of k^* that will minimize the maximum bending moment in the beam.

3.6.4 Homework # 4

Solve the following problem in the class notes: 5.5.5. **Due date:** Feb. 22^{nd} 2008.

3.6.5 Homework # 5

Solve the following problems in the class notes: 6.7.2 and 6.10.4. **Due date:** Feb. 29^{th} 2008.

3.6.6 Homework # 6

Solve the following problems in the class notes: 7.1.1, 7.1.4, 7.2.3, 7.3.1, 7.5.2. **Due date:** Mar. 14^{th} 2008.

3.6.7 Homework # 7

Solve the following problems in the class notes: 8.4.5, 8.6.1. **Due date:** Apr. 4^{th} 2008.

3.6.8 Homework # 8

Solve the following problems in the class notes: 11.1.2 and 11.1.13. **Due date:** Apr. 21^{st} 2008.

4 AE-6107: Analysis of Aerospace Structural Elements

Here is where to find more information about

- The mandatory homework format, see section 1.
- The statement of academic honesty, see section 2.
- Lectures: M-W-F 13:05 to 13:55, in Engineering Science & Mechanics Building, Room G8.

4.1 Topics to be covered in this course

The following topics to be covered in this course:

1. *Euler-Bernoulli beam theory.* Review of the basic assumption.
2. *Torsion of beams.* Saint Venant's solution.
3. *Yield criteria.* Introduction to yield criteria. Plastic behavior of trusses. Plastic bending and torsion of beams.
4. *Thermoelasticity.* Introduction to the behavior of structures under thermal loading.
5. *Kirchhoff plate theory.* Theory for isotropic plates. Decoupling into bending and stretching equations. Boundary conditions. Energy methods.
6. *Anisotropic plates.* Classical lamination theory. Governing equations for anisotropic plates.
7. *Classical solutions for plates.* The Navier solution; the Lévy solution. Energy methods for plates.
8. *Mindlin plate theory.* Shearing deformations in plates. Navier solution for shear deformable plates. Energy principles for shear deformable plates.
9. *Von Karman equations.* Governing equations for plates undergoing large displacements and rotations.
10. *Buckling of anisotropic plates.* Compressive and shearing loads. Effects of shearing deformations.
11. *Vibrations of plates.* Effects of shearing deformations and in-plane loads.

4.2 Grading Policy

The overall numerical grade for this course will be computed using the weighting factors shown in table 3.

8-10 Homework Assignments	40%
4 Exams	40%
1 Final exam	20%
TOTAL	**100%**

Table 3: Grade weighting factors

4.3 Weekly homework

Homework will be assigned on a weekly basis. Homework is assigned on Fridays lectures and is due the next Friday, in class. Homework is a vital part of the learning process, **and 40% of your final grade.** To make sure no homework is *forgotten*, an aging factor will be built into the grading as shown in table 4. Do not forget to use the mandatory homework format, see section 1.

Date homework is turned in	**Aging factor**
On the due date	actual grade
Up to the Monday after due date	actual grade - 1/10
Up to the Wednesday after due date	actual grade - 2/10
Never turned in	0/10

Table 4: Aging factor for homework

4.4 Quizzes and Final

Four one-hour quizzes will be held at the dates posted on the schedule below. Quizzes are closed book and closed notes, but open minds. Three crib sheets, hand written front and back, are allowed at each quiz. The comprehensive final exam will be held on Apr. 28th 14:50 - 17:40. The format of the final is identical to that of the quizzes, but five crib sheets are allowed.

4.5 Reference books

The following reference text books are a good source of information for the class: [2, 7, 4, 5, 8, 9, 10, 11, 12, 13].

4.6 Course Schedule

Week	Dates	Reading Assignments	Homework
1	Jan. 7th - 11th	Chapters 1 and 5	
2	Jan. 14th - 18th	Chapters 2 and 7	Hw # 1, see sec. 4.7.1
3	Jan. 21st - 25th	Chapters 16	**Quiz # 1:** Jan. 25th
4	Jan. 28th - Feb. 1st	Chapters 16	Hw # 2, see sec. 4.7.2
5	Feb. 4th - 8th	Chapters 16	Hw # 3, see sec. 4.7.3
6	Feb. 11th - 15th	Chapters 14	Hw # 4, see sec. 4.7.4
7	Feb. 18th - 22nd	Chapters 14	
8	Feb. 25th - 29th	Chapters 14	Hw # 5, see sec. 4.7.5
9	Mar. 3rd - 7th	Chapters 8	**Quiz # 3:** Mar. 7th
10	Mar. 10th - 14th	Chapters 8	
11	Mar. 17th - 21st		Enjoy the **Spring Break**
12	Mar. 24th - 28th	Chapters 8	Hw # 6, see sec. 4.7.6
13	Mar. 31st - Apr. 4th	Chapters 15	**Quiz # 4:** Apr. 4th
14	Apr. 7th - 11th	Chapters 15	Hw # 7, see sec. 4.7.7
15	Apr. 14th - 18th		Hw # 8, see sec. 4.7.8
16	Apr 21st - 25th		**Final**: Apr. 28th 14:50 - 17:40

4.7 Homework problems

Do not forget to use the mandatory homework format, see section 1.

4.7.1 Homework # 1

Solve problems 1.3.7, 1.6.5, 5.5.5 and 5.5.7 in the notes. **Due date:** Jan. 21st 2008.

4.7.2 Homework # 2

Solve problems 16.4.1, 16.4.2 in the notes. **Due date:** Feb. 8th 2008.

4.7.3 Homework # 3

Solve the three problems below. **Due date:** Feb. 15th 2008.

Consider a bi-cantilevered beam subject to a parabolic temperature distribution. This problem is similar to that depicted in fig. 3, except that the beam is now cantilevered at both ends. *(1)* Find the axial and transverse deflection of the beam under the thermal loading. *(2)* Find the thermal stress distribution in the beam.

Consider the bi-cantilevered beam with a sudden change in cress-section geometry, as depicted in fig 4. The left portion of the beam is of length L_1 and the rectangular cross-section has a width b and height h_1, whereas the corresponding dimensions for the right portion of the beam are L_2, b and h_2, respectively. Both portions of the beam are subjected to a parabolic thermal gradient, as indicated on the figure. *(1)* Find the axial and transverse deflection

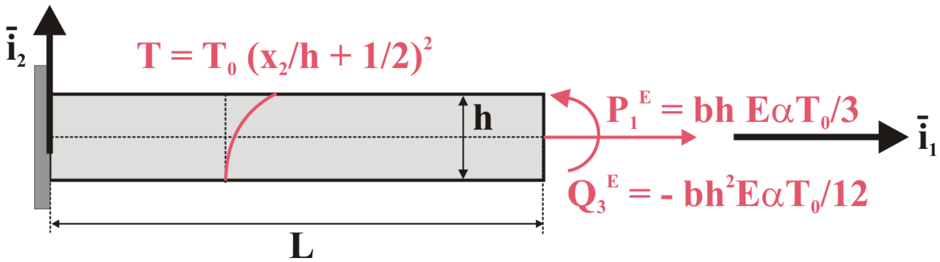

Figure 3: Cantilevered beam under parabolic thermal gradient.

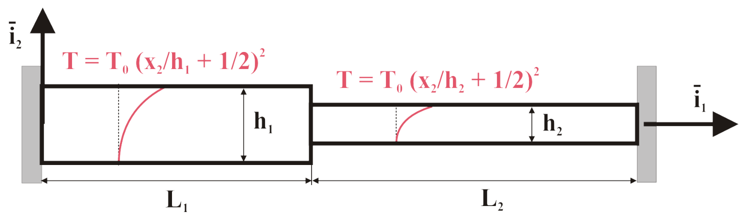

Figure 4: Bi-cantilevered beam under parabolic thermal gradients.

fields for the beam under the thermal loading. *(2)* Find the thermal stress distributions in the beam.

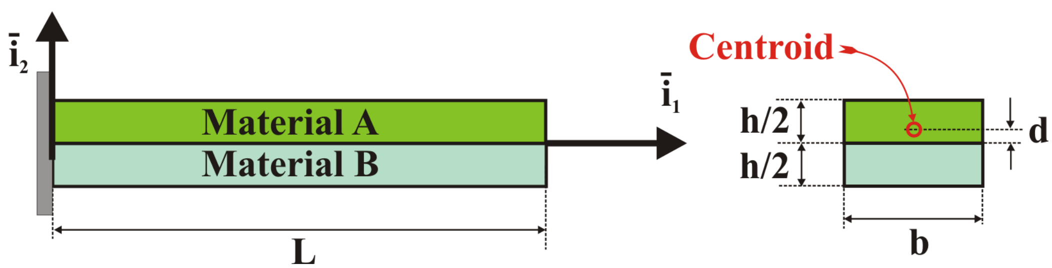

Figure 5: Cantilevered bi-material beam under uniform temperature field.

Consider the cantilevered beam of length L made of two half-beam, each of width b and height $h/2$, welded together along the beam's mid-plane, as depicted in fig. 5. The top beam is made of a material of Young's modulus E_a and coefficient of thermal expansion α_a, whereas the corresponding quantities for the lower beam material are E_b and α_b, respectively. The entire assembly is heated to a uniform temperature T. *(1)* Find the transverse displacement field for the bi-material beam. *(2)* On a single graph, plot the transverse displacement field for the six combinations of materials chosen from the materials listed in table 5. *(3)* What is the best combination of materials if the beam's tip deflection per degree of heating is to be maximized? *(4)* Find the thermal stress distribution in the beam. *(5)* On a single graph, plot the axial stress distribution over the cross-section the six combinations of materials chosen from the materials listed in table 5. *(6)* Find the location of the maximum axial stress. For what combination of materials are the thermal stresses maximized? *(7)* Check

Material	Coefficient of thermal expansion [μ/C]	Young's modulus [GPa]
Aluminum	23	73
Copper	17	120
Steel	11	210
Titanium	8.6	110

Table 5: Coefficients of thermal expansion and Young's moduli for commonly used structural materials.

that the thermal stress field is a self equilibrating stress field.

4.7.4 Homework # 4

Solve the two problems below. **Due date:** Feb. 22$^{\text{nd}}$ 2008.

In the notes, thermal effects in beams were studied using the constraint method. Show that thermal effects in beams can also be studied using the direct method applied to Euler-Bernoulli beam theory for beams under axial and transverse loads, respectively. Show that the two approaches are equivalent.

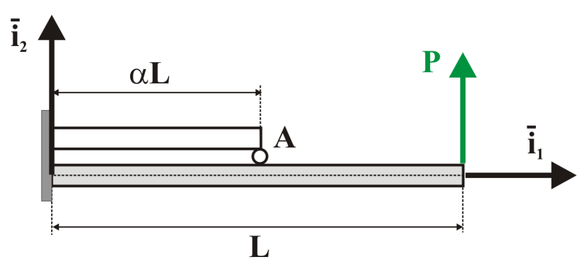

Figure 6: Cantilevered beam with intermediate support.

The cantilevered beam depicted in fig. 6 is subjected to a tip load P. The tip of a second cantilevered beam contacts the first at point **A**. The lower and upper beams have a uniform bending stiffness, H_{33}^c, uniform shearing stiffnesses, K_{22}, and are of length L and αL, respectively. *(1)* Find the displacement fields for the two beams. *(2)* Plot the distribution of transverse displacement, bending moment, and shear force for both beams. Use $\alpha = 0.25$. For each question, consider the following cases: *(a)* the beam has no shearing deformations (*i.e.* assume Euler-Bernoulli beam theory); *(b)* the beam is made of steel and has a rectangular cross-section with $E/G = 2.6$ and $h/L = 1/5$, see section A in fig. 14.10; *(c)* the beam has a sandwich cross-section with $E_f/G_c = 35$, $t/h = 1/10$, $h/L = 1/5$, see section B in fig. 14.10. *(3)* Study the behavior of the system as $\alpha \to 0$. Comment on your results.

4.7.5 Homework # 5

Solve the problem below. **Due date:** Mar. 15$^{\text{th}}$ 2008.

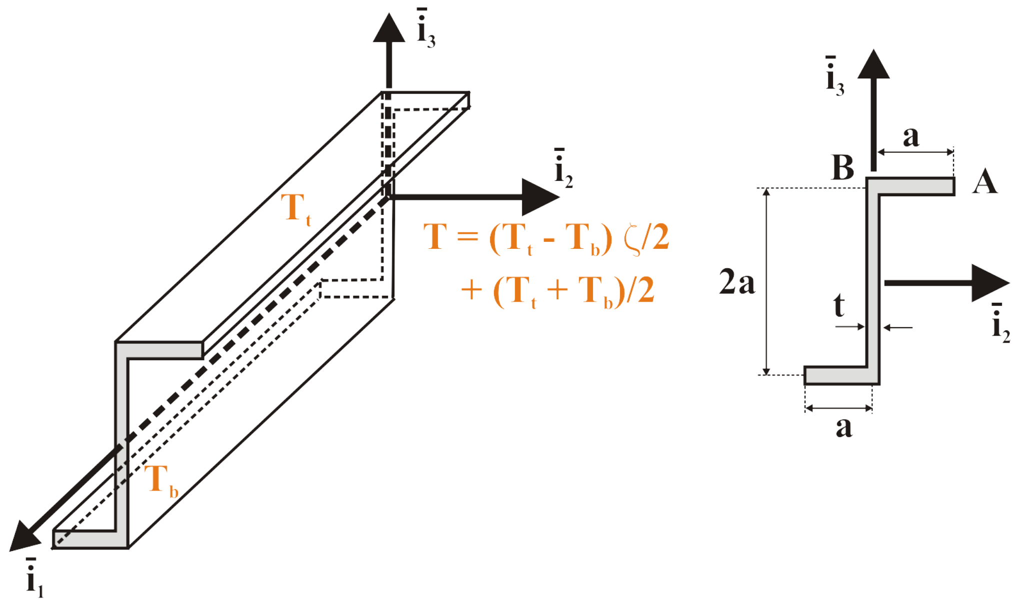

Figure 7: Cantilevered beam subjected to thermal gradient.

In class, the governing equations for beams with symmetric cross-sections were derived based on the direct method and Euler-Bernoulli kinematic assumptions. Generalize the governing equations for beam with unsymmetric cross-sections developed in chapter 6 to the case where such beams are subjected to arbitrary thermal gradients. Note that for such problems the kinematic description given in section 6.1 remains unchanged, and the equilibrium equations of the problem, see section 6.3, are still valid as well. However, the sectional constitutive laws derived in section 6.2 must be updated to accommodate thermal effects. *(1)* Derive the governing equations when principal centroidal axes of bending are used. *(2)* Derive the governing equations when centroidal axes are used that are not aligned with the principal axes of bending.

Consider now the problem of a cantilevered beam with a "Z" section as treated in example 6.9.1. Figure 7 shows the cantilevered beam subjected to the following temperature field: the top and bottom flanges are at temperatures T_t and T_b, respectively, whereas the temperature profile in the vertical web is $T = (T_t - T_b)\zeta/2 + (T_t + T_b)/2$, where $\zeta = x_3/a$. *(3)* Find the tip deflections of the beam. *(4)* Determine and plot the axial stress distribution over the cross-section of the beam. Use the following data: $T_t = \lambda T_b$, $\lambda = 3$.

4.7.6 Homework # 6

Solve problem 8.6.1 and 8.9.1 in the notes. **Due date:** Apr. 4^{th}.

4.7.7 Homework # 7

Solve problems 15.2.1, 15.2.2, 15.2.3 and 15.2.4. **Due date:** Apr. 14^{th} 2008.

4.7.8 Homework # 8

Solve problem 15.3.1, 15.3.2 and 15.3.3 in the notes. **Due date:** Apr 21st 2008.

5 AE-6210: Advanced Dynamics

Here is where to find more information about

- The mandatory homework format, see section 1.
- The statement of academic honesty, see section 2.
- Lectures: M-W-F 11:00 to 12:00, Guggenheim Building, Room 244.

5.1 Topics to be covered in this course

The following topics to be covered in this course:

1. Vectors and tensor. Scalar, vector and tensor products. Second order tensors.
2. Coordinate systems. Differential geometry of curves, surfaces and three dimensional mappings.
3. Basis principles of dynamics. Newton's laws for particles and systems of particles.
4. The geometric description of rotations. Euler angles. Time and spatial derivatives.
5. Kinematics of rigid bodies.
6. Kinetics of rigid bodies.
7. Variational and Energy principles in dynamics.

5.2 Grading Policy

The overall numerical grade for this course will be computed as follows:

10 Homework Assignments	50%
3 Quizzes	30%
1 Final exam	20%
TOTAL	**100%**

5.3 Weekly homework

Homework will be assigned on a weekly basis. Homework will be assigned on the Friday lecture and is due the next Friday, in class. This homework is a vital part of the learning process, **and 50% of your final grade.** To make sure no homework is *forgotten* an aging factor will be built into the grading as follows

Date homework is turned in	Aging factor
On the due date	actual grade
Up to the Monday after due date	actual grade - 1/10
Up to the Wednesday after due date	actual grade - 2/10
Never turned in	0/10

Do not forget to use the mandatory homework format, see section 1.

5.4 Reference books

The following reference text books are a good source of information for this class and are on reference in the library. [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

5.5 Course Schedule

Week	Dates	Reading Assignments	Homework
1	Aug. 21st - 25th	Chapter 1	
2	Aug. 28th - Sep. 1st	Chapter 1 & 2	
3	Sep. 4th - 8th	Chapter 2	
4	Sep. 11th - 15th	Chapter 3	**Quiz # 1:** Sep. 13th
5	Sep. 18th - 22nd	Chapter 3 & 4	
6	Sep. 25th - 29th	Chapter 4	
7	Oct. 2nd - 6th	Chapter 4	
8	Oct. 9th - 13th	Chapter 5	**Quiz # 2:** Oct. 11th
9	Oct. 16th - 20th	Chapter 5 and 6	Oct 16-17th, **Fall recess**
10	Oct. 23rd - 27th	Chapter 5	
11	Oct. 30th - Nov. 3rd	Chapter 7	
12	Nov. 6th - 10th	Chapter 7	**Quiz # 3:** Nov. 8th
13	Nov. 13th - 17th	Chapter 8	
14	Nov. 20th - 24th	Chapter 8	**Happy Thanksgivings**
15	Nov. 27th - Dec. 1st	Chapter 8	
16	Dec. 4th - 8th		
17	Dec. 11th - Dec. 15th		**Final**:

References

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