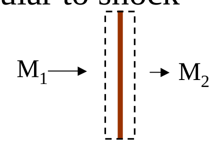
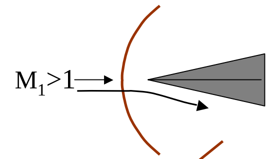
Supersonic Flow Turning

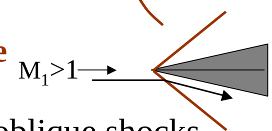
- For normal shocks, flow is perpendicular to shock
  – no change in flow direction



- How does supersonic flow change direction, i.e., make a turn
  – either slow to subsonic ahead of turn (can then make gradual turn) =**bow shock**



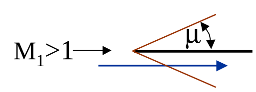
  – go through non-normal wave with sudden angle change , i.e., **oblique shock** (also expansions: see later)



- Can have straight/curved, 2-d/3-d oblique shocks
  – will examine straight, 2-d oblique shocks

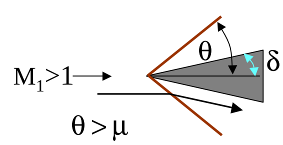
Oblique Shock Waves

- Mach wave
  – consider infinitely thin body
  – no flow turn required
  – infinitessimal wave

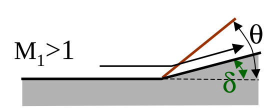


$$\mu = \sin^{-1}(1/M_1)$$

- Oblique shock
  – consider finite-sized wedge, half-angle, δ
  – flow must undergo compression
  – if attached shock
    ⇒ oblique shock at angle θ
  – similar for concave corner



$\theta > \mu$



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Equations of Motion

- Governing equations
  - same approach as for normal shocks
  - use conservation equations and state equations
- Conservation Equations
  - mass, energy and momentum
  - this time 2 momentum equations - 2 velocity components for a 2-d oblique shock
- Assumptions
  - steady flow (stationary shock), inviscid except inside shock, adiabatic, no work but flow work

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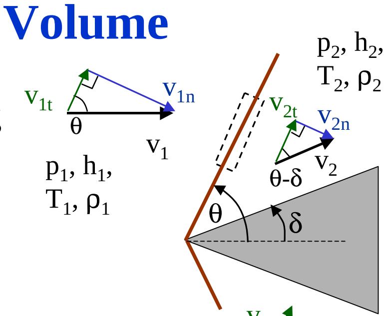
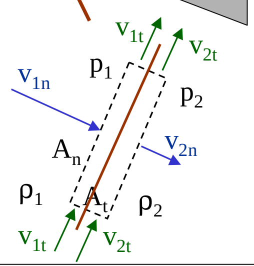
Control Volume



- Pick control volume along shock
- Divide velocity into two components
  - one tangent to shock, **v_t**
  - one normal to shock, **v_n**
- Angles from geometry
  - $v_{1n}=v_1\sin\theta$; $v_{1t}=v_1\cos\theta$
  - $v_{2n}=v_2\sin(\theta\text{-}\delta)$; $v_{2t}=v_2\cos(\theta\text{-}\delta)$
  - $M_{1n}=M_1\sin\theta$; $M_{1t}=M_1\cos\theta$
  - $M_{2n}=M_2\sin(\theta\text{-}\delta)$; $M_{2t}=M_2\cos(\theta\text{-}\delta)$



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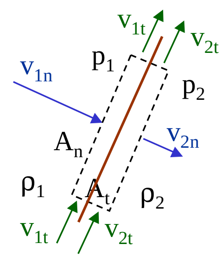
Conservation Equations

- **Mass** $\quad 0=\int_{CS}\rho(\vec{v}_{rel}\cdot\vec{n})dA$

$$\rho_1 v_{1n}A_n+\cancel{\rho_1 v_{1t}\frac{A_t}{2}}+\cancel{\rho_2 v_{2t}\frac{A_t}{2}}=$$
$$\rho_2 v_{2n}A_n+\cancel{\rho_1 v_{1t}\frac{A_t}{2}}+\cancel{\rho_2 v_{2t}\frac{A_t}{2}}$$

$$\rho_1 v_{1n}=\rho_2 v_{2n} \qquad \textbf{(1)}$$



- **Momentum** $\quad -\int_{CS}p\cdot\vec{n}\,dA=\int_{CS}\rho\vec{v}(\vec{v}_{rel}\cdot\vec{n})dA$

tangent $\quad \cancel{(p_1+p_2)\frac{A_t}{2}-(p_1+p_2)\frac{A_t}{2}}=v_{1t}(-\cancel{\rho_1 v_{1n}A_n})+v_{2t}(\cancel{\rho_2 v_{2n}A_n})$

$$v_{1t}=v_{2t}$$

normal $\quad p_1A_n-p_2A_n=v_{1n}(-\rho_1 v_{1n}A_n)+v_{2n}(\rho_2 v_{2n}A_n)$

$$p_1-p_2=\rho_1 v_{1n}^2+\rho_2 v_{2n}^2 \qquad \textbf{(2)}$$

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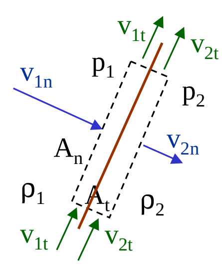
Conservation Equations (con't)

- **Energy**

$$\cancel{\rho_1 v_{1n}}A_n\left(h_1+\frac{v_1^2}{2}\right)=\cancel{\rho_2 v_{2n}}A_n\left(h_2+\frac{v_2^2}{2}\right)$$

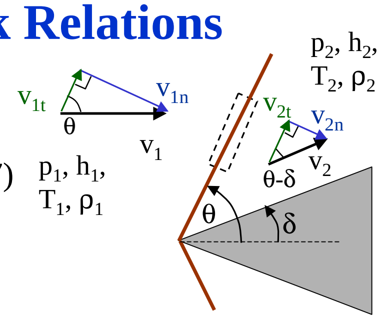
$$h_1+\frac{v_{1n}^2}{2}+\cancel{\frac{v_{1t}^2}{2}}=h_2+\frac{v_{2n}^2}{2}+\cancel{\frac{v_{2t}^2}{2}}$$

$$h_1+\frac{v_{1n}^2}{2}=h_2+\frac{v_{2n}^2}{2} \qquad \textbf{(3)}$$



- Eq's. (1)-(3) are same equations used to characterize normal shocks (VII.1-3) with $v_n \rightarrow v$
- So oblique shock acts like normal shock in direction normal to wave
  - v_t constant, but $M_{t1}\neq M_{t2}$ $\quad \frac{M_{t2}}{M_{t1}}=\frac{\overset{1}{\cancel{v_{t2}}}/a_2}{\cancel{v_{t1}}/a_1}\Rightarrow \frac{M_{t2}}{M_{t1}}=\sqrt{\frac{T_1}{T_2}}$ **(VII.20)**

Oblique Shock Relations

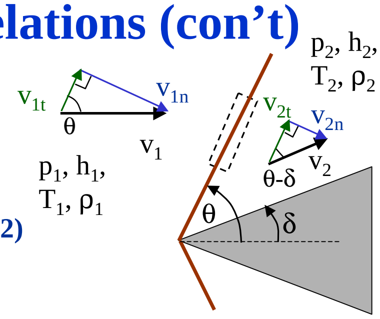


- To find conditions across shock, use M relations from normal shocks, e.g., (VII.5-17) but replace
  $M_1 \rightarrow M_1 \sin\theta$
  $M_2 \rightarrow M_2 \sin(\theta\text{-}\delta)$
- **Mach Number**

from (VI.31) $$M_2^2 = \left(M_1^2 + \frac{2}{\gamma-1}\right)\bigg/\left(\frac{2\gamma}{\gamma-1}M_1^2 - 1\right)$$

$$M_2^2 \sin^2(\theta-\delta) = \left(M_1^2 \sin^2\theta + \frac{2}{\gamma-1}\right)\bigg/\left(\frac{2\gamma}{\gamma-1}M_1^2 \sin^2\theta - 1\right) \quad \textbf{(VII.21)}$$

Oblique Shock Relations (con't)



- **Static Properties**

(from VII.12)
$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1}M_1^2 \sin^2\theta - \frac{\gamma-1}{\gamma+1} \quad \textbf{(VII.22)}$$

(from VII.18)
$$\frac{v_{1n}}{v_{2n}} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2 \sin^2\theta}{(\gamma-1)M_1^2 \sin^2\theta + 2} \quad \textbf{(VII.23)}$$

(from VII.9)
$$\frac{T_2}{T_1} = \frac{\left(1 + \frac{\gamma-1}{2}M_1^2 \sin^2\theta\right)\left(\frac{2\gamma}{\gamma-1}M_1^2 \sin^2\theta - 1\right)}{M_1^2 \sin^2\theta \, (\gamma+1)^2 \big/ 2(\gamma-1)} \quad \textbf{(VII.24)}$$

Oblique Shock Relations (con't)

- **Stagnation Properties**

T_o (from energy conservation)

$$T_{o2} = T_{o1}$$

p_o (from VII.14) $\quad p_{o2}/p_{o1} = (T_1/T_2)^{\gamma/\gamma-1}(p_2/p_1)$

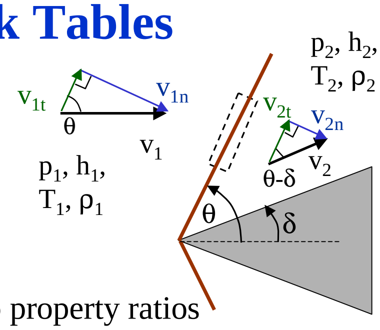
since function of static property ratios, don't have to factor in p_{ot} v. p_{on}

(from VII.13)

$$\frac{p_{o2}}{p_{o1}} = \left[\frac{\frac{\gamma+1}{2}M_1^2\sin^2\theta}{1+\frac{\gamma-1}{2}M_1^2\sin^2\theta}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{2\gamma}{\gamma+1}M_1^2\sin^2\theta-\frac{\gamma-1}{\gamma+1}\right]^{\frac{1}{1-\gamma}}$$

(VII.25)

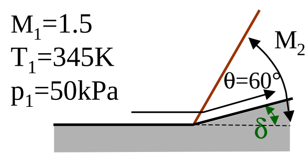
Use of Shock Tables



- Since just replacing
  $M_1 \rightarrow M_1\sin\theta$
  $M_2 \rightarrow M_2\sin(\theta\text{-}\delta)$
  - can also use normal shock tables
  - use $M_1'=M_1\sin\theta$ to look up property ratios
  - $M_2= M_2'/\sin(\theta\text{-}\delta)$, with M_2' from normal shock tables
- **Warning**
  - do not use p_1/p_{o2} from tables
  - only gives p_{o2} associated with v_{2n}, not v_{2t}

Example #1

- **Given:** Uniform Mach 1.5 air flow (p=50 kPa, T=345K) approaching sharp concave corner. Oblique shock produced with shock angle of 60°



- **Find:**
  1. T_{o2}
  2. p_2
  3. δ (turning angle)
- **Assume:** TPG/CPG with γ=1.4, steady, adiabatic, no work, inviscid except for shock,….

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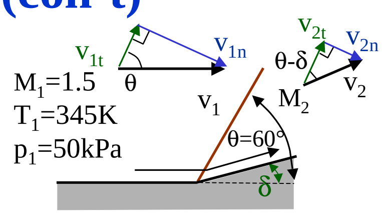
Example #1 (con't)

- **Analysis:**
  - T_o

$$T_{o2} = T_{o1} = T_1\left(1+\frac{\gamma-1}{2}M_1^2\right)$$
$$= 345\text{K}\left(1+0.2(1.5)^2\right) = \boxed{500\text{K}}$$



  - p_2
    - calculate normal component

$$M_{1n} = M_1 \sin\theta = 1.5\sin 60^\circ = 1.30 \qquad \text{(B.1)} \Rightarrow M_{2n} = 0.786$$
$$p_2/p_1 = 1.805$$
$$T_2/T_1 = 1.191$$
$$p_2 = (p_2/p_1)p_1 = (1.805)50\text{kPa} = \boxed{90.3\text{kPa}}$$

  - δ

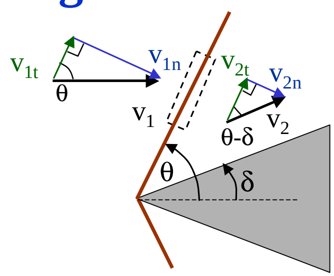
(from VII.20)

$$\tan(\theta-\delta) = M_{2n}/M_{2t} = M_{2n}\Big/\left(M_{1t}\sqrt{T_1/T_2}\right) = M_{2n}\Big/\left(M_1\cos\theta\sqrt{T_1/T_2}\right)$$
$$\delta = 60^\circ - \tan^{-1}\left(\frac{0.786}{1.5\cos 60^\circ \times 1.191^{-0.5}}\right) = \boxed{11.2^\circ}$$

NOTE: $M_2 = M_{2n}/\sin(\theta-\delta) = 1.04 > 1$ Supersonic flow okay after oblique shock

Wave/Shock Angle

- Generally, wave angle θ is not a given, rather know turning angle δ
- Find relationship between M_1, δ, and θ



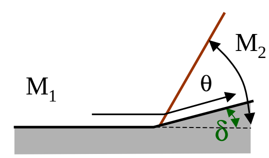
(from VI.43) (from geometry)

$$\frac{v_{1n}}{v_{2n}} = \frac{(\gamma+1)M_1^2\sin^2\theta}{(\gamma-1)M_1^2\sin^2\theta+2} = \frac{\cancel{v_{1t}}\tan\theta}{\cancelto{1}{v_{2t}}\tan(\delta-\theta)}$$

$$\boxed{\tan\delta = \frac{(2/\tan\theta)\left(M_1^2\sin^2\theta-1\right)}{M_1^2(\gamma+\cos 2\theta)+2}} \qquad \text{(VII.26)}$$

So to find oblique shock solution, need **2 indep. parameters**, e.g., M_1 and δ

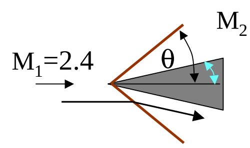
Oblique Solution Summary



- If given $\mathbf{M_1}$ and turning angle, **δ**
  1. Find θ from (iteration) VII.26 or use oblique shock charts (e.g., Appendix C in John)
  2. Calculate $M_{1n}=M_1\sin\theta$
  3. Use normal shock tables or Mach relations, e.g., VII.22-25 to get property ratios
  4. Get M_2 from $M_2=M_{2n}/\sin(\theta-\delta)$ or VII.21

Example #2

- **Given:** Uniform Mach 2.4, cool, nitrogen flow passing over 2-d wedge with 16° half-angle.



- **Find:**

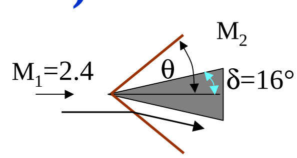
  θ, p_2/p_1 , T_2/T_1 , p_{o2}/p_{o1} , M_2

- **Assume:** N_2 is TPG/CPG with γ=1.4, steady, adiabatic, no work, inviscid except for shock,….

Example #2 (con't)

- **Analysis:**
  - θ (from VII.26)

$$\tan 16^\circ = \frac{(2/\tan\theta)(2.4^2 \sin^2\theta - 1)}{2.4^2(1.4 + \cos 2\theta) + 2}$$



iterate $\theta = 39.4^\circ$

  - **use shock relations** calculate normal component

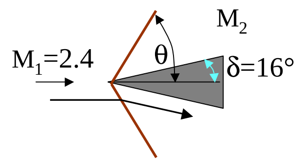
$$M_{1n} = M_1 \sin\theta = 2.4 \sin 39.4^\circ = 1.52$$

$$\text{(B.1)} \Rightarrow M_{2n} = 0.6935;\ p_2/p_1 = 2.535;\ T_2/T_1 = 1.335;\ p_{o2}/p_{o1} = 0.9227$$

$$M_2 = M_{2n}/\sin(\theta - \delta) = 1.75$$ Supersonic after shock

Example #2 (con't)

- **Analysis (con't):**
  - **a second solution for θ**



$$\tan 16^\circ = \frac{(2/\tan\theta)\left(2.4^2\sin^2\theta - 1\right)}{2.4^2(1.4+\cos 2\theta)+2}$$

in addition to 39.4° $\theta = 82.1^\circ$

  - **use shock relations** calculate normal component

$$M_{1n} = M_1 \sin\theta = 2.4\sin 82.1^\circ = 2.38$$

previous solution **2.535** **1.335** **0.9227**

$$\text{(B.1)} \Rightarrow M_{2n} = 0.5256;\ p_2/p_1 = 6.425;\ T_2/T_1 = 2.018;\ p_{o2}/p_{o1} = 0.5499$$

1.75

$$M_2 = M_{2n}/\sin(\theta-\delta) = 0.575$$ **Now subsonic after shock**

- VII.26 generally has **2 solutions for θ**: **Strong** and **Weak** oblique shocks