Practice Problems on Shearing Flows

1. Thwaites' method & Karman integral method

1.1 In 1938 Howarth studied flows with a linearly decelerating external velocity distribution:

$$u_e(x) = A\left\{1 - \frac{x}{L}\right\}$$

Using Thwaites' method, show that the separation point will occur at x/L ≅0.123.

1.2 Apply Thwaites' method to flow over a circular cylinder. Note that the external velocity distribution may be given by

$$u_e(x) = 2V_\infty \sin\left(\frac{x}{R}\right)$$

where x is the distance from the front stagnation point, measured along the circumference of the cylinder, and R is the cylinder radius.

Determine where the flow will separate.

Hint: You will need to know solution to the integral $\int \sin^5\theta \cdot d\theta$

1.3 Prandtl recommended that the velocity profile for turbulent boundary layer flows over a flat plate be approximated by

$$\frac{u}{V_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

Use this velocity profile in Von Karman-Pohlhausen technique (i.e. solution integral momentum equation) to show that the boundary layer thickness varies with x as

$$\frac{\delta}{x} \approx \frac{0.16}{\mathrm{Re}_x^{1/7}}$$

where Re_x is the Reynolds number based on x. Show also that the skin friction coefficient $C_f(x)$ is given by

$$C_f(x) \approx \frac{0.027}{\mathrm{Re}_x^{1/7}}$$

and that the shape factor is approximately 1.3.

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2. Sample Mid-Quarter Test #1

Closed Book Portion

2.1. Briefly describe the following terms.

a) Specific Internal Energy, e
b) Specific Enthalpy, h
c) Stagnation Enthalpy, h_0
d) Prandtl Number, Pr

10%

2.2 Consider a face ABCD of a cubical control volume in a flow:

Name and draw all the viscous stresses acting on this face, with the correct direction, and correct subscripts.

10%

2.3 Briefly describe what we mean by a Newtonian fluid. Give an example of a Newtonian fluid, and an example of a non-Newtonian fluid.

10%

2.4 Briefly describe what physical processes cause viscous stresses and heat conduction in gases.

10%

2.5 Briefly define the term "Reynolds Number". Describe how it relates to the inertial forces and viscous on a fluid element.

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Open Book part

2.6. Consider steady, laminar incompressible viscous flow through a pipe of radius R shown. At the entrance the velocity is uniform, given by U. At the exit, the velocity is given by $C[R^2 - r^2]$.

From a control volume analysis of the conservation of mass and u- momentum, show that the drag force experienced by the pipe is given by $\pi R^2(\Delta p - \rho U^2 / 3)$ where Δp is the pressure drop across the length of the pipe.

Hint: a) Compute the constant C by equating mass inflow rate with the mass outflow. Note that the mass outflow rate is given by

$$\dot{m} = \rho \int_0^R 2\pi r \cdot u(r) dr$$

15%

b) Find the momentum inflow rate and the momentum outflow rate. Note that the momentum outflow rate is given by

$$\dot{m} = \rho \int_0^R 2\pi r \cdot u^2 dr$$

15%

c) Compute the pressure forces on the left and right faces of the control volume, assuming uniform pressure p and $p+\Delta p$ on these two faces.

5%

d) Apply momentum equation which states that the time rate of change of momentum within the control volume equals

Momentum inflow rate - momentum outflow rate + Pressure forces acting on C.V. + Viscous Drag forces acting on C.V.

15%

Solve for the Drag Force.

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3. Sample Midterm Test #2: Open Book and Notes

3.1. Given the transformation that Blasius used

$$\psi = \sqrt{\nu V_\infty x} \cdot f(\eta)$$

$$\eta = \frac{1}{2}\sqrt{\frac{V_\infty}{\nu x}} \cdot y$$

Derive expressions for u , v , ∂u/∂x and ∂v/∂y. Is continuity satisfied?

20%

3.2. Hiemenz used the following relations to study stagnation point flow.

$$\psi = \sqrt{\nu u_e x} \cdot f(\eta)$$

$$u_e = Ax \quad \text{and} \quad \eta = \sqrt{\frac{u_e}{\nu x}} \cdot y$$

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = -u_e \frac{\partial u_e}{\partial x} = -A^2 x$$

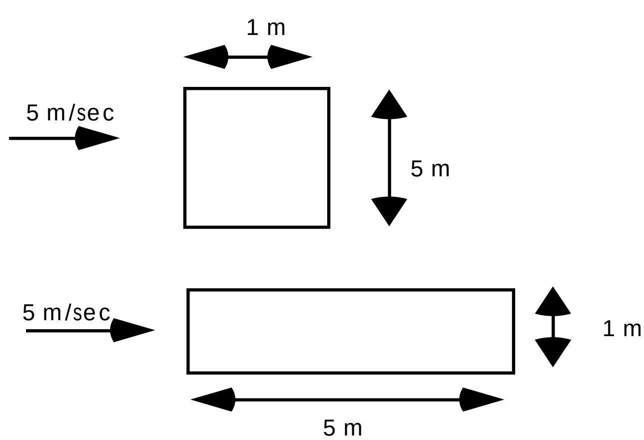
Show that these relations, when used in the u- momentum equation, lead to the following ordinary differential equation for the function f(η).

$$\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} + 1 - \left\{\frac{df}{d\eta}\right\}^2 = 0$$

Hint: Plug in the definition of u_e = Ax into stream function and η before you do anything else; You will also find ∂η/∂x = 0 .

40%

3.3. Evaluate the drag coefficients of the two flat plates shown, placed at zero angle of attack, in a low speed air flow at sea level conditions. Use strip theory.



Which of these two has a higher drag coefficient? Why? 40%