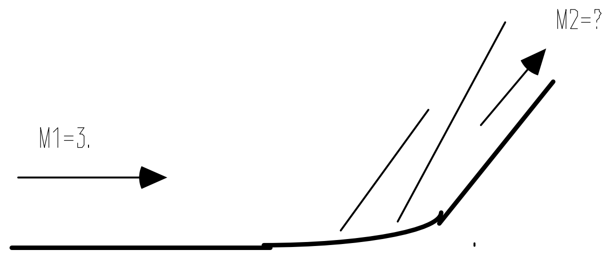
High Speed Aerodynamics

Typical Problems (Many are from Prof. Sankar's notes)

1. Review of Gasdynamics

1.1

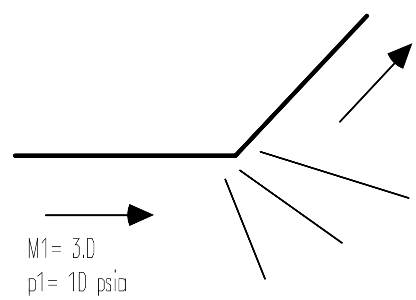


Assuming that the flow turns by (a) 10° and (b) 30°,

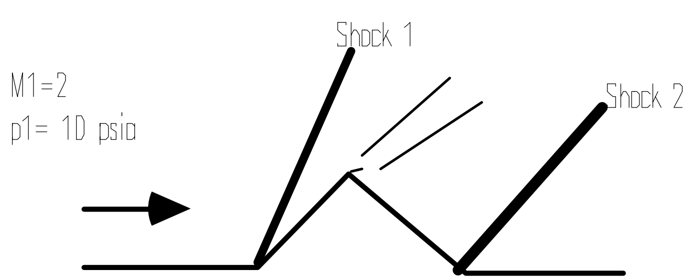
(i) Find M_2 based on isentropic compression theory (Ans: 2.527, 1.767)

(ii) Find M_2 based on oblique shock theory (Ans: 2.51, 1.41)

1.2



Assuming that the flow turns by 20°, find M_2 , p_2 and p_{02}.
(Ans: 4.319, 1.595 psia, 367.33 psia).3.



Determine p2, p3, po4 (Ans: 28.5 psia, 2.995 psia, 58.45 psia).

2. Linearized Potential Equation; Method of Characteristics

2.1. Consider the linearized potential flow equation:

$$\left(1 - M_\infty^2\right)\phi_{xx} + \phi_{yy} = 0$$

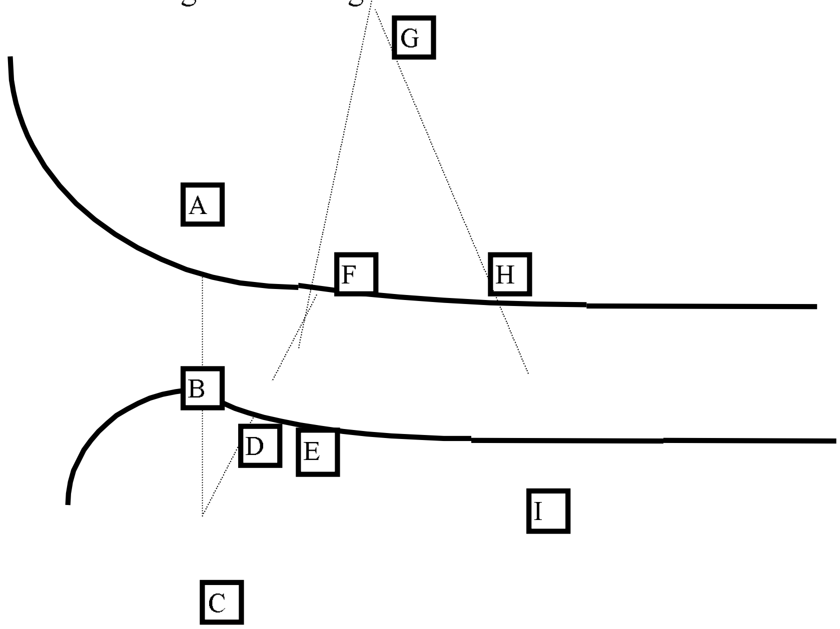
a) Show that this equation is elliptic (i.e. has no real characteristics) if M_∞ is less than 1.

b) Show that this equation is hyperbolic (i.e. has real characteristic lines) if M_∞ is greater than 1, and that the characteristic equation is given by

$$\frac{dy}{dx} = \pm\frac{1}{\sqrt{M_\infty^2 - 1}}$$

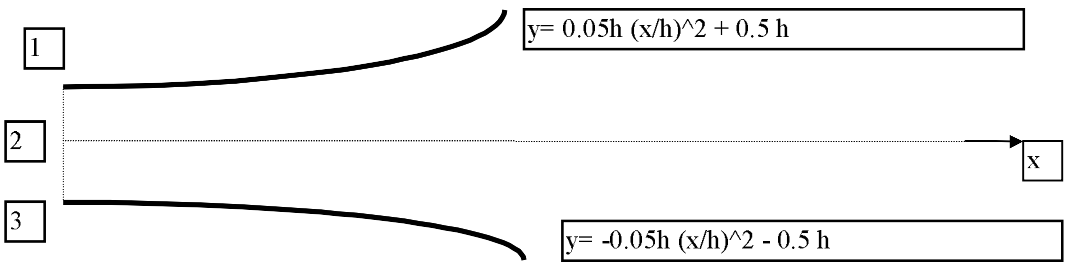
c) When M_∞ is greater than 1, derive a compatibility relation linking ϕ_x and ϕ_y.

2.2. A nozzle is designed to deliver a parallel, uniform stream of air at a Mach number of 2.059. The general arrangement of the nozzle is shown in the sketch below:



At the inlet (Line AB) the flow is parallel to the x- axis, at a Mach number of unity.
In the region AFDB, the flow is to be modeled as a corner flow, with the corner centered at the fictitious point C. The nozzle segments BD and AF become streamlines associated with this corner flow. This corner flow produces one half of the expansion needed. That is, ν at the end of this expansion is $\nu_{exit}/2$.
In the region FDE, the flow is uniform, and parallel.
In the region EFHI, the flow is to be modeled as a corner flow, with the corner centered at a fictitious corner G. The segments EI and FH become streamlines associated with this corner flow. This segment provides the rest of the expansion, raising the Prandtl-Meyer angle from $\nu_{exit}/2$
to ν_{exit}.
Downstream of the line HI, the flow is uniform, horizontal and has an exit Mach number of 2.05. Design this nozzle, and sketch it on a graph paper, or using Excel.

2.3. The 2-D passage sketched has parabolically shaped walls. At the inlet section 1,2,3 the flow is uniform and parallel at a Mach number of 2.03. Using the points 1, 2 and 3 as the starting points, compute the pressure distribution along the nozzle wall, for an axial distance 4h downstream of the entrance, where h is the inlet height.



3. Subsonic Potential Flow: Transformations

3.1. Show that the transformation

$$\xi = \frac{x}{\beta};\ \eta = y,\ \varphi_2 = C\varphi$$

where,

$$\beta = \sqrt{1 - M_\infty^2}$$

will transform the equation

$$\beta^2 \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$$

to

$$\frac{\partial^2 \varphi_2}{\partial \xi^2} + \frac{\partial^2 \varphi_2}{\partial \eta^2} = 0$$

3.2. For a wind tunnel test conducted at a compressible flow Mach number of 0.34, the lift curve slope of a 2-D airfoil was measured to be 6.38. Use the Prandtl-Glauert similarity rule to find the lift curve slope of the same airfoil at a freestream Mach number of 0.67.

Ans: 8.08

3.3. Experimental data at 40% chord on the upper surface of a NACA 0010 airfoil tested at very low speeds at 4 degree angle of attack was Cp = - 0.56. What would be the static pressure (psia) at 40% chord on the upper surface for the same airfoil at M_∞ = 0.5 at the same angle of attack? Assume $p_{0\infty}$ to be 2500 psfa.

Ans: 13 psia

3.4. A NACA 0016 airfoil at a = 0 degrees has a certain pressure distribution in a compressible flow at M_∞ equal to 0.6. I wish to test a NACA 0018 airfoil at zero angle of attack in a second tunnel. What should the M_∞ be for this second tunnel to have an identical Cp distribution?

Ans: 0.436

Hint: NACA 0018 airfoil has a slope that is 1.5 times that of NACA 0012 airfoil. You need to show that the Cp distributions of these two airfoils are related by this factor 1.5, at a given Mach number.

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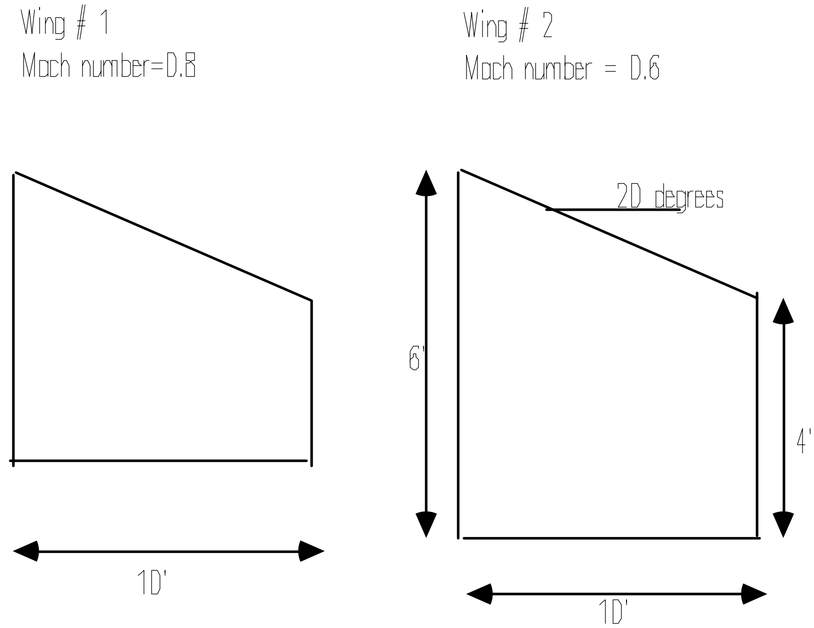
5. Subsonic Wing Aerodynamics

5.1. Consider subsonic linearized potential flow past the two wings shown:



The two wings use identical airfoil families along the entire span, the same twist distribution, and operate at the same angle of attack. The two wings have identical span, equal to 10 feet. The two wings operate at freestream Mach numbers 0.8 and 0.6 respectively.

Determine

a) The root chord, the tip chord and the leading edge sweep angle for wing #1, for these two wings to be related by a similarity transformation.

b)The relation linking the pressure distribution C_p for the two flows, at identical spanwise and non-dimensional chordwise locations.

c)The relationship linking the total lift coefficient C_L for the two wings.

2. An untwisted wing of elliptical planform has an aspect ratio of 10 and is tested at Mach 0.7. Assuming that the airfoil sections have an incompressible lift curve slope of 2□, determine $dC_L/d\alpha$ for this wing at Mach 0.7.

Ans: 6.87

Hint: You need Incompressible Wing Theory given by Anderson, Pages 331-334. In particular, for an elliptical lift distribution, $C_l = 2\pi(\alpha - \alpha_i)$ is constant along the span in incompressible flows. You will find the incompressible wing $dC_L/d\alpha$ is 4.91

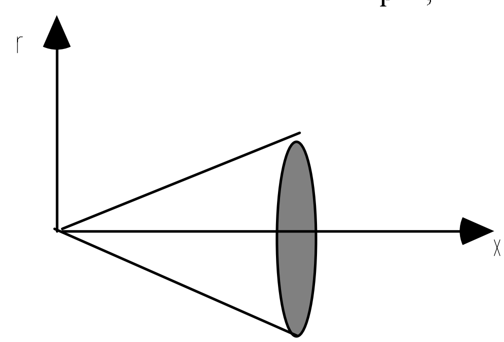
6. Bodies of Revolution

6.1. A number of wind tunnel models were constructed for a series of tests. All were bodies of revolution, affinely related to each other, but having different values of the ratio of maximum diameter (L/D). The purpose of the tests was to illustrate subsonic similarity.
For the L/D = 10 model tested at M =0.4 (compressible), the suction peak was Cp = - 2.0.

a)At what Mach number should the model with L/D= 6 be tested to illustrate similarity?

b)What will be the suction peak for the L/D= 8 model, when the freestream Mach number is 0.68 ?

6.2. The supersonic potential flow over a cone shown is modeled through a distribution of "supersonic sources" along its axis. Determine the source strength distribution f(x) in terms of the semi-vertex angle δ and the distance from the cone apex, x.



8. EXERCISES

8.1. Two wind tunnel models were constructed for a series of tests. These were bodies of revolution, affinely related to each other, but having different values of the ratio of maximum diameter (L/D). The purpose of the tests was to verify Gothert's rule - i.e. their Cp distributions differ from each other only by a constant, if the freestream Mach numbers are chosen correctly.

The first model had an L/D of 10, while the second had an L/D of 6.For the L/D = 10 model tested at M =0.4 (compressible), the suction peak was Cp = - 2.0.

a) At what Mach number should the model with L/D= 6 be tested to apply Gothert's rule?

b) What will be the suction peak for the L/D= 6 model?

2. Consider two similar 3-D wings of rectangular planform in subsonic flows. The wing in compressible flow has chord "c", span "b" and aspect ratio "AR". The wing in incompressible flow is described by chord c_0, span b_0 and aspect ratio AR_0. The two flows are related by the transformation:

$$x = \xi \; ; \; \eta = \beta y \; ; \qquad \zeta = \beta z \; ; \qquad \varphi = \beta^5 \varphi_0$$

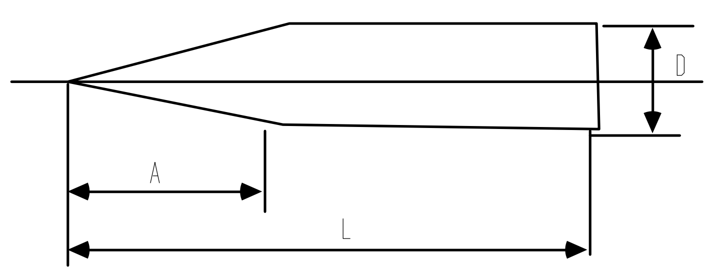
Find (a) $C_p = (?)\, C_{p0}$, (b) $AR = (?)\, AR_0$

3. An untapered, untwisted wing of aspect ratio 4.0, sweep angle 20 degrees is flying at 0.85 freestream Mach number, at 5 degree angle of attack. The wing uses NACA 0008 airfoil sections at all span stations.

a) Find the aspect ratio, sweep angle, airfoil section and aspect ratio of an equivalent wing (i.e. a wing that obeys similarity) of the same span, to be tested in an incompressible flow wind tunnel.

b) If the 2-D lift curve slope is 0.1 per degree for the NACA 0008 sections tested in a wind tunnel, what is the 2-D lift curve slope for the Mach 0.85 wing?

4. Consider axi-symmetric supersonic flow past the cone-cylinder missile configuration shown below:



This flow is to be solved using a distribution of supersonic "sources" along the body axis.

a) Determine the source strength f(x) along the body axis, $0 < x < L$.
b) Find the velocity potential j(x,r) for the two regions, $0 < x < A$ and $A < x < L$
c) Evaluate and sketch the Cp distribution for the entire body length $0 < x < L$ for the following case: Freestream Mach number = 3.0, D= 0.25 meters, A = 1 meter, L = 3 meters.

Solutions

1 a) 0.835
b) -3.056

2. a) $C_p = \beta^5 C_{p,0}$
b) $AR = \frac{1}{\beta} AR_0$

3. a) AR of incompressible wing is 2.10713
Λ of incompressible wing is 34.64 degrees
Same airfoil section, same angle of attack (Do you know why?)
b) 0.1898

4. a) For $0 < x < A$, $f(x) = V_\infty (\tan^2 \delta) x \approx V_\infty \delta^2 x$ where δ is the cone semi-vertex angle, assumed to be small .
For $x > A$, $f(x) = 0$

b)

For $0 < x < A$:

$$S(\xi) = \pi R^2 = \pi \delta^2 \xi^2$$

$$S'(\xi) = 2\pi\delta^2 \xi$$

$$S''(\xi) = 2\pi\delta^2$$

$$\varphi(x,r) = -V_\infty x \delta^2 \ln\left(\frac{2}{\lambda r}\right) - V_\infty \delta^2 \int_{\xi=0}^{\xi=x} \ln(x-\xi)\, d\xi$$

You may leave this integral in this form. <u>You need not evaluate the integral</u>, although it is not difficult, if you use the indefinite intgeral relation

$$\int \ln(x)\, dx = x\ln(x) - x$$

The final expresion would be

$$\varphi(x,r) = -V_\infty x \delta^2 \ln\left(\frac{2}{\lambda\delta}\right) + V_\infty x \delta^2$$

For $x > A$:

$$\varphi(x,r) = -V_\infty \delta^2 \int_{\xi=0}^{\xi=A} \ln(x-\xi)\, d\xi = -V_\infty \delta^2 [A \ln(x-A) - A]$$

Again, you need not integrate this expression.

c) These need not be done, since they require eavlaution of the integrals above, followed by computation of δφ/ δ x. Again, in case you are interested,

0 < x < A: $C_p = 2\delta^2 \ln\left(\frac{2}{\lambda\delta}\right) - 3\delta^2$

A < x < L $C_p = \frac{2\delta^2 A}{x - A}$ Notice the singularity at x=A, a sharp corner.