High Speed Aerodynamics

Problem Sets from Previous Courses (courtesy Prof. Sankar)

1. 2-D Compressible Potential Flow

(These problems extend the 2-D compressible potential flow equation derived in the class to 3-D, and to axi-symmetric flows. You should be able to use an approach identical to what we did in two-dimensions.)

1.1 Given the continuity equation:

$$\frac{\partial\left(\rho\frac{\partial\phi}{\partial x}\right)}{\partial x}+\frac{\partial\left(\rho\frac{\partial\phi}{\partial y}\right)}{\partial y}+\frac{\partial\left(\rho\frac{\partial\phi}{\partial z}\right)}{\partial z}=0$$

the energy equation:

$$C_pT+\frac{\phi_x^2+\phi_y^2+\phi_z^2}{2}=Const$$

and the isentropic gas law:

$$T=C\rho^{\gamma-1}\quad \text{where C is a constant,}$$

Derive the 3-D form of the compressible potential flow equation given below:

$$\left(a^2-\phi_x^2\right)\phi_{xx}+\left(a^2-\phi_y^2\right)\phi_{yy}+\left(a^2-\phi_z^2\right)\phi_{zz}-2\phi_x\phi_y\phi_{xy}-2\phi_z$$

2. Transformations for Subsonic Aerodynamics

2.1 A NACA 0015 airfoil was tested at zero angle of attack in a low speed wind tunnel. The minimum value of the surface pressure coefficient was equal to –0.66, and was found to occur near 30% chord.
Determine where on the airfoil the flow will first turn sonic. Determine the critical Mach number. Ans: M_{crit} = 0.67

2.2 The potential flow velocity distribution over a circular cylinder is given by:

$$V = 2\ V_\infty \sin\theta$$

Where θ is the angular position of any point on the circular cylinder, measured from the front stagnation point. Determine the critical Mach number for this body.
Ans: 0.404

2.3 Problem 11.2 in Anderson's text. Ans= 0.938
2.4 Problem 11.4 in the text. Ans= -1.53

2.5 Problem 11.5 in the text. Ans = 0.805
2.6 Problem 11.7 in the text.

3. Sample Midterm Test

3.1 List the assumptions that we made to arrive at the compressible potential flow equations.

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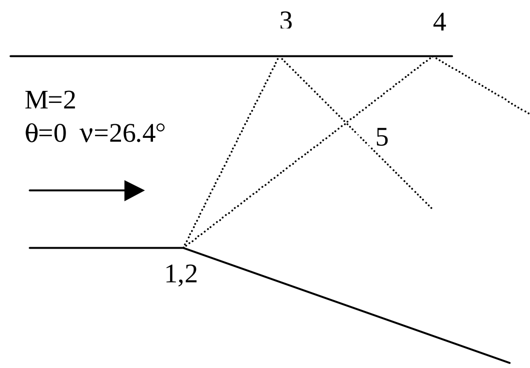
3.2 Consider the flow through a 2-D divergent duct shown below. Upstream of the corner, the flow is a Mach number of 2, and parallel to the x-axis. On the bottom wall, there is a sharp corner that turns the flow by 10 degrees as shown. This turn is approximated by two discrete turns at points 1 and 2, each equal to 5 degrees. Thus, $\theta_1 = -5°$ and $\theta_2 = -10°$. (Note the negative sign!)



Determine the flow angle θ and the Prandtl-Meyer angle ν at the points 3, 4 and 5.

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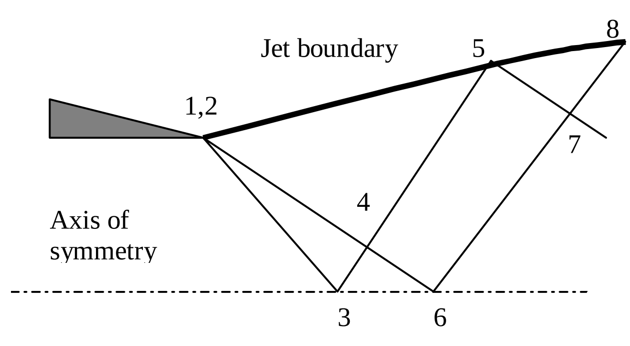
3.3. Consider an under-expanded jet exhausting into atmosphere. At the nozzle exit, the Mach number is 2.0 and the flow is parallel to the x- axis. The exit pressure is 1.1 times the ambient pressure.
At the nozzle lip the flow expands to the atmospheric pressure through an expansion fan. Assume that this expansion takes place in two distinct waves, originating at points 1 and 2. Each of these two waves turns the flow by an equal amount.



Determine the flow angle θ and the Prandtl-Meyer angle ν at points 3,4,5,6,7 and 8.

Mach Number		Prandtl-Meyer Angle		p_0/p
1.5		11.90193008		3.671031

1.55		13.37752922		3.948475
1.6		14.85626105		4.250414
1.65		16.33338794		4.578863
1.7		17.80500489		4.935993
1.75		19.26788439		5.324135
1.8		20.71935647		5.745796
1.85		22.15721434		6.203668
1.9		23.57963894		6.700636
1.95		24.98513793		7.239792
2		26.37249571		7.824449
2.05		27.74073208		8.45815
2.1		29.08906783		9.144683
2.15		30.41689578		9.888097
2.2		31.7237563		10.69271
2.25		33.0093165		11.56314
2.3		34.27335244		12.50428
2.35		35.51573386		13.52139
2.4		36.73641099		14.62002
2.45		37.93540318		15.8061
2.5		39.11278902		17.08594
2.55		40.2686978		18.46621
2.6		41.40330198		19.95403
2.65		42.51681072		21.55692
2.7		43.60946414		23.28287
2.75		44.68152841		25.14035
2.8		45.73329134		27.1383
2.85		46.76505859		29.2862
2.9		47.77715032		31.59408

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4. Sample Midterm # 2

Closed Book and Notes, 50 Minutes

4.1.State the assumptions made in deriving the equation:

$$\boxed{\left(1 - M_\infty^2\right)\varphi_{xx} + \varphi_{yy} = 0}$$

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4.2 Define/explain the following:

a) Prandtl-Glauert Rule
b) Critical Mach number

20%

4.3 The incompressible potential flow over a circular cylinder yields the following velocity distribution at the surface:

$$V = 2\, V_\infty \sin\theta$$

Where θ is the angular position of any point on the circular cylinder, measured from the front stagnation point.

Determine the critical Mach number for this body.

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4.4 Sketch the variation of the lift and drag coefficient C_l and C_d as a function of the freestream Mach number in the subsonic, transonic, and the supersonic regimes. Explain why the loads vary as shown.

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Some relationships you may or may not need:

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\frac{p}{p_\infty} - 1}{\frac{\gamma}{2} M_\infty^2} = \frac{\left(\frac{\rho}{\rho_\infty}\right)^\gamma - 1}{\frac{\gamma}{2} M_\infty^2}$$

Isentropy:

$$\frac{p}{\rho^\gamma} = \text{const}$$

$$\frac{T}{\rho^{\gamma-1}} = \text{const}$$

Energy Equation:

$$C_p T + \frac{|\overline{V}|^2}{2} = h + \frac{|\overline{V}|^2}{2} = \frac{a^2}{\gamma - 1} + \frac{|\overline{V}|^2}{2} = \text{const}$$

where,

$$\overline{V} = \overline{\nabla}\phi$$