

AE 3051, Lab #16

**Investigation of the Ideal Gas State Equation**

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## Abstract

The validity of the ideal gas equation of state was experimentally tested for three gases: helium (He), argon (Ar) and nitrogen (N<sub>2</sub>). The experiments were conducted in a fixed volume container over a range of pressures (~30-60 kPa) and temperatures (~295-360 K). Gas temperatures were determined with a type-K thermocouple, and the pressures were measured with a mercury manometer. The measurements for each gas were obtained with a single, fixed gas sample; thus each sample had a constant mass. In agreement with the ideal gas law for these conditions, the density ratio,  $p/RT$ , was found to be constant, at least within the ~5% uncertainty of the measurements. The uncertainty in  $p/RT$  was primarily a result of the repeatability of the pressure and temperature conditions, which could be improved by more carefully ensuring that steady-state is achieved. In addition, limitations in the apparatus did not allow the mass of each sample to be measured. Therefore, the value of the ideal gas constant,  $R$ , could not be determined.

## Introduction

An important relationship often used to relate specific gas properties is the ideal gas equation of state. This report describes experiments intended to investigate the validity of this state equation, which can be written as  $p = \rho RT$ , where  $p$  is the absolute pressure,  $T$  is the absolute temperature and  $R$  is the specific gas constant. In order to verify the ideal gas state equation, the pressure of a fixed gas sample was measured as it was heated in fixed volume container. Three individual gases: helium (He), nitrogen (N<sub>2</sub>) and argon (Ar), were tested. The temperature and pressure inside the container were measured with a thermocouple and a mercury manometer. In addition, the volume of the container was determined by measuring an equivalent volume of water.

## Instrumentation and Procedure

Figure 1 illustrates the experimental apparatus employed in this investigation. The fixed-volume, aluminum test cell is composed of a cylindrical body and two end flanges. There are three connections to the cell: the first connects the cell to a control valve and to the high pressure cylinder containing the test gas; the second connects to another control valve and a vacuum pump; and the third connects the cell to a vertical-type mercury manometer. The test cell tubing attaches to the top of the vertical arm of the manometer, while the mercury bath is open to the atmosphere. Mounted vertically, alongside the manometer, is a ruler, which has a fine scale marked in 1/32" units. The apparatus also includes a digital barometer for measuring the local atmospheric pressure in the room.

Electrical-resistance heating tape wrapped around the cell controls the cell's temperature, as well as the temperature of the gas inside. The voltage across the heating tape, and therefore the

current through it, is regulated by a variable transformer. A type-K thermocouple passes through the side of the cell and monitors the temperature of the gas inside. The thermocouple leads connect to a calibrated electronic readout unit that includes an electronic reference junction. In order to ensure that the gas temperature is nearly uniform inside the test cell, the cell is covered with insulation. In addition, there is a stirring mechanism at the bottom of the cell that is magnetically-coupled to a motor outside the cell.

In order to measure the volume of the cell, the top flange was removed and the cell was filled with water. The water was then emptied into a 100 ml graduated cylinder. After drying the cell carefully, the flange was replaced and tightened. Then the vacuum pump was turned on for a period of 20 minutes so as to empty the cell of air. For each test gas, the following measurement procedure was employed. First, the cell was evacuated with the vacuum pump to a gauge pressure below 29.7 in. Hg. Then, the cell was pressured to ~1 atm with the test gas (at approximately room temperature). This cycle of filling and evacuating was repeated three more times to ensure the cell's contents were limited to the test gas of interest. Then, the test cell was filled to a pressure below 1 atm, the magnetic stirrer was activated, and the thermocouple readout was monitored until it did not change by more than 0.1°C in 10 seconds. With the container at steady-state, the pressure and temperature readings were acquired. Next, the voltage on the heating tape was then increased a small amount, and the pressure and temperature measurements were taken again after the thermocouple readout stabilized.

## **Results and Discussion**

As noted above, the test cell's volume was ascertained by filling the test cell with water and measuring the water volume with the graduated cylinder. This process was repeated a total of 10

times and the results are shown in Table I. The average of the ten measurements is 83 ml, or equivalently, 83 cm<sup>3</sup>. Next, the lab group measured the atmospheric pressure using the digital barometer. The measured pressure was 29.66 in. Hg.

Initially, six measurements of cell gauge pressures as a function of temperature were taken for each gas. The results are listed in Table II for all three gases (He, N<sub>2</sub> and Ar). The measured temperatures and pressures can be converted to absolute values using the following conversions:

$$T(K) = T(^{\circ}C) + 273 \quad (1)$$

$$p(Pa) = [p_{gage}(in.Hg) + p_{atm}(in.Hg)] \frac{101325Pa}{760mmHg} \frac{25.4mmHg}{1in.Hg} \quad (2)$$

With the measured atmospheric pressure in the test room, Equation (2) reduces to:

$$p(Pa) = 3386[p_{gage}(in.Hg) + 29.66] \quad (3)$$

The raw data listed in Table II were converted to absolute values using Equations (1) and (3). They are shown plotted in Figure 2. For all three gases, the data shown in Figure 2 appear to fall along a single line in  $p,T$  space. This agrees with the ideal gas state equation, i.e.,  $p=constant \times T$ , which is the equation for a line going through the origin.

An alternate way to compare the results to the ideal gas law is to look at the slope of the line, i.e.,  $p/T$ , or even better, the ratio  $p/RT$ . The mass specific gas constant  $R$  is given by:

$$R = \bar{R} / \bar{M} \quad (4)$$

where  $\bar{R}$  is the universal gas constant (8314 J/kmol K) and  $\bar{M}$  is the molecular weight of the gas (see Table III). The results for  $p/RT$  as a function of temperature are plotted in Figure 3, and the actual values are listed in Table IV. Since the gas inside the container is trapped and the container volume is fixed (assuming negligible change in the size of the container due to thermal

expansion during heating), the measurements for each gas sample are at the same density. According to the ideal gas state equation,  $\rho = p/RT$ . So, we would expect the ratio  $p/RT$  to remain constant in our experiment as the gas temperature is changed. This is essentially the behavior displayed in Table IV and Figure 3. In addition since all the gases were contained in the same volume, we would expect that the measured  $p/RT$  values would scale like the molecular weight of each gas. This is exactly the behavior seen in Figure 4, which shows the average  $p/RT$  measurement of each gas (see Table IV) as a function of molecular weight.

In order to test the repeatability of the measurements, one temperature setting for the last test gas (Ar) was repeated six more times (see Table V). The measurement was repeated by changing the cell temperature away from 39°C, then returning it, and finally measuring the pressure. This complete cycle was repeated for each measurement. As covered below in the Error Analysis section, the repeatability in the ratio  $p/RT$  was ~5%. This is about the same value as that found for all the  $p/RT$  results.

## **Supplemental Question Discussion**

### Fluid Dynamics

With the current measurements, we are not able to find the value for the universal gas constant. Another way to write the ideal gas equation of state is  $pV = mRT$ . In the current experiments, we measured the pressure, volume and temperature, but not the mass of the gas in the test cell. So, we can not determine  $R$  as we have only one equation but two unknowns ( $m$  and  $R$ ).

### Experimental Techniques

In order to test the remainder of the ideal gas law, one would have to determine the mass of gas trapped inside the cylinder with some independent measurement. One possibility would be to

weigh the test cell twice: once with a vacuum inside and once when it is loaded with the test gas. The difference would be the weight of the test gas, which could be converted to mass using the standard value for the gravitational acceleration on Earth. We could then compare this to the mass determined from combining: 1) the  $p$  and  $T$  measurements, 2) the ideal gas law, and 3) the volume of the test cell, i.e.,

$$m = \frac{P}{RT}V \quad (5)$$

However, getting an accurate value with Equation (5) would be a very difficult task. Assuming the ideal gas law is accurate, our density results from this experiment for the heaviest gas, Ar, were approximately  $0.49 \text{ kg/m}^3$ . Since the volume of the test cell was measured to be  $83 \text{ cm}^3$ , the mass of the Ar inside the cell was only 41 mg. It would be difficult to measure this small change in the mass of the test cell, which was approximately 2000g. In other words, an error of 0.1% in the measurement of the “empty” test cell would be  $\sim 2\text{g}$ . To increase the mass of gas to the level of our expected uncertainty in the test cell’s mass, the initial pressure in the cell would need to be increased from  $\sim 80 \text{ kPa}$  (see Figure 2) to roughly 2.4 MPa or almost 350 psi. In addition, it would probably require a stronger *and heavier* chamber to withstand this high pressure, making the measurement even more difficult.

In the current experiment, we kept the density of the gas constant and changed its temperature. An alternate approach would be to use a piston-cylinder arrangement to change the volume and, therefore, the density of the gas and measure the pressure and temperature. The drawback to this approach is that the volume would need to change very rapidly and the temperature measurement would need to be quick enough to prevent errors due to heat exchange with the cylinder walls. In other words, if one were to rapidly compress the gas, its temperature

would rise above the temperature of the cylinder walls. Very quickly, however, the gas would start losing heat to the walls and the temperature would drop back towards its original value.

## Error Analysis

### Volume

The volume of the test cell was measured 10 times (see Table I). The random error or precision of the volume measurement,  $u_{V,prec}$ , can be estimated from the standard deviation of the repeated volume measurements. From the measured values, this is 0.28 ml. Assuming a normal distribution, the random error within a 95% confidence interval is  $u_{V,prec} = \pm 0.55$  ml ( $\pm 1.96 \times 0.28$  ml). One source of systematic or bias error,  $u_{V,bias}$ , in the measurement is the resolution of the graduated cylinder, which is 0.5 ml. Assuming the graduated cylinder is accurately calibrated, this is likely the largest error associated with the cylinder itself. Another source of systematic error in the measurement is the water left in the test cell after it is emptied into the graduated cylinder. Based on a visual observation, this is estimated to be less than 15 drops ( $< 2$  mm diam.) of water or less than 0.06 ml. This can be neglected compared to the resolution error. Thus the combined or total error, as given by

$$u_i = \sqrt{u_{i,prec}^2 + u_{i,bias}^2} \quad (6)$$

is  $u_V = 0.74$  ml. As a percentage of the average value of the measured volume,  $u_V / \bar{V} = 0.9\%$ .

### Density

The density measurements ( $p\bar{M} / \bar{R}T$ ) are affected by errors in measurements of the pressure and temperature, as well as the uncertainty in the purity of the gas in the test cell, which influences the molecular weight  $\bar{M}$ . Since the lab group did not make repeated measurements of

the temperature or gas sample, we can only estimate the systematic errors of these two measurements.

The manufacturer specifies the accuracy of the thermocouple that we used as better than 0.1 °C, i.e.,  $u_{T,bias}=0.1$  °K. With regard to the gas purity, we can estimate the amount of residual gas (nontest gas) left in the cell by considering the refilling process. Each time the cell was evacuated to a pressure below  $-29.7$  in. Hg gage ( $\sim 670$  Pa) and then filled to 1 atm with test gas, the residual gas was diluted to 0.66% ( $=670/101325$ ) of the test gas. Since the dilution process was repeated four times, the residual gas should compose less than  $\sim 10^{-9}$  (1 part per billion) of the gas in the cell during the test. Our worst case error will be the presence of air in the He test gas (a heavy gas in the lightest test gas). So the bias error can be estimated from the relationship

$$u_{\bar{M},bias} = \left| \bar{M}_{actual} - \bar{M}_{He} \right| , \quad (7)$$

and inserting the appropriate values into Eq. (7),

$$\begin{aligned} u_{\bar{M},bias} &= \left| \left( (1 - 10^{-9}) \bar{M}_{He} + 10^{-9} \bar{M}_{Air} \right) - \bar{M}_{He} \right| \\ &\cong -10^{-9} (28.8 - 4) \\ &= 2.5 \times 10^{-8} \end{aligned}$$

The random error in the pressure measurement can be estimated from the repeated Ar measurements (Table V). Including the original measurement at 39 °C (Table II), the standard deviation of the pressure measurements is 804 Pa. Again using a 95% confidence interval, we can estimate  $u_{p,prec} \approx \pm 1580$  Pa. The primary systematic error in the manometer is the resolution of the device, which is 1/32 in. Hg or  $u_{p,bias} = \pm 106$  Pa. The total pressure error, from Equation (6), is  $u_p = \pm 1584$  Pa, which is essentially the repeatability error in the pressure measurement.

We can combine all these errors into a single error in density measurement by using the relation,

$$u_{\rho} = \sqrt{\left(\frac{\partial \rho}{\partial T} u_T\right)^2 + \left(\frac{\partial \rho}{\partial \bar{M}} u_{\bar{M}}\right)^2 + \left(\frac{\partial \rho}{\partial p} u_p\right)^2} \quad (8)$$

The partial derivatives required in Eq. (8) are obtained from the ideal gas state equation, so that

$$\begin{aligned} u_{\rho} &= \sqrt{\left(-\frac{p\bar{M}}{RT^2} u_T\right)^2 + \left(\frac{p}{RT} u_{\bar{M}}\right)^2 + \left(\frac{\bar{M}}{RT} u_p\right)^2} \\ &= \rho \sqrt{\left(-\frac{1}{T} u_T\right)^2 + \left(\frac{1}{\bar{M}} u_{\bar{M}}\right)^2 + \left(\frac{1}{p} u_p\right)^2} \end{aligned}$$

Inserting the uncertainty values given above for the repeated Ar case, we find

$$\begin{aligned} \frac{u_{\rho}}{\rho} &= \sqrt{\left(\frac{0.1K}{312K}\right)^2 + \left(\frac{2.5 \times 10^{-8}}{40}\right)^2 + \left(\frac{1584Pa}{31550Pa}\right)^2} \\ &= \sqrt{\left(3.2 \times 10^{-4}\right)^2 + \left(6 \times 10^{-10}\right)^2 + \left(5.0 \times 10^{-2}\right)^2} \\ &= 0.05 \end{aligned}$$

Thus, the relative error in the current measurements of density is ~5%. Clearly, the most significant error is the repeatability in the pressure measurements. This relative error was used to produce the error bars shown in Figure 3. It agrees with the general repeatability of the density measurements for the Ar case, as well as other gases listed in Table IV. The most likely source of this repeatability error is the amount of time that the lab group waited for the system to come to steady-state. It is likely that the thermocouple measurement of the cell temperature was not representative of the actual gas temperature, because the temperature was not uniform inside the cell. One way to reduce this error would be to wait longer before taking data.

## Conclusions

This report describes an experiment to verify the ideal gas equation of state for He, N<sub>2</sub> and Ar. Specifically, the ratio  $p/RT$  was calculated based on pressure and temperature measurements for fixed mass samples in a fixed volume container. Since the density of the sample could not change in the fixed volume container, the ideal gas equation would predict that the ratio  $p/RT$  would be constant. For each test gas sample, the measured density was found to vary by less than  $\sim\pm 5\%$  as the temperature of the cell was varied over a range of roughly 295-360 K. Also, the variation appears to be random. In other words, it does not appear to be a function of the pressure and temperature. Based on attempts to repeat a single point, most of the variation in the measurements can be attributed to lack of repeatability in reproducing the pressure and temperature conditions. Increased attention to ensuring that the cell had reached steady-state would likely reduce this error. In addition, the results showed that the density of the gas samples varied directly with the molecular weight of the gas as would be expected from the ideal gas law. So even though a specific value for the universal gas constant could not be determined from the reported data, the current results are sufficient to validate the ideal gas state equation, at least within the range of pressures and temperatures tested.

## Figures and Tables

Table I. Test cell volume results determined from equivalent volume of water.

<b>Sample #</b>	<b>Volume (ml)</b>
1	83.0
2	83.0
3	82.5
4	83.0
5	83.0
6	83.5
7	83.0
8	82.5
9	83.0
10	83.0

Table II. Pressures and temperatures for 3 gases in a heated, fixed volume container.

<b>Gas</b>	<b>T (°C)</b>	<b>p<sub>gage</sub> (in. Hg)</b>
He	22	20 13/32
He	29	20 15/32
He	38	20 4/32
He	50	19 28/32
He	65	19 18/32
He	85	19 7/32
N <sub>2</sub>	23	21 5/32
N <sub>2</sub>	28	20 23/32
N <sub>2</sub>	40	19 29/32
N <sub>2</sub>	49	19 22/32
N <sub>2</sub>	63	19 28/32
N <sub>2</sub>	81	19 2/32
Ar	22	20 31/32
Ar	30	20 19/32
Ar	39	20 21/32
Ar	51	19 17/32
Ar	63	19 19/32
Ar	84	19 4/32

Table III. Molecular weights of the gases measured.

Gas	$\bar{M}$ (kg/kmol)
He	4
N <sub>2</sub>	28
Ar	40

Table IV. Measurements of the ratio  $p/RT$ ; also listed are the average value of the ratio for each gas.

Gas	T (K)	$p/RT$ (kg/m <sup>3</sup> )
He	295	0.0511
He	302	0.0496
He	311	0.0499
He	323	0.0493
He	338	0.0487
He	358	0.0475
<b>He</b>	<b>Average</b>	<b>0.0493</b>
N <sub>2</sub>	296	0.328
N <sub>2</sub>	301	0.339
N <sub>2</sub>	313	0.355
N <sub>2</sub>	322	0.353
N <sub>2</sub>	336	0.332
N <sub>2</sub>	354	0.342
<b>N<sub>2</sub></b>	<b>Average</b>	<b>0.341</b>
Ar	295	0.478
Ar	303	0.488
Ar	312	0.470
Ar	324	0.509
Ar	336	0.488
Ar	357	0.481
<b>Ar</b>	<b>Average</b>	<b>0.486</b>

Table V. Repeated values of pressure and temperature for Ar at 39°C (312 K).

Repeat	$p_{\text{gage}}$ (in. Hg)	$p/RT$ (kg/m <sup>3</sup> )
1	20 5/32	0.496
2	20 17/32	0.477
3	20 16/32	0.478
4	20 4/32	0.498
5	20 12/32	0.485
6	20 21/32	0.470

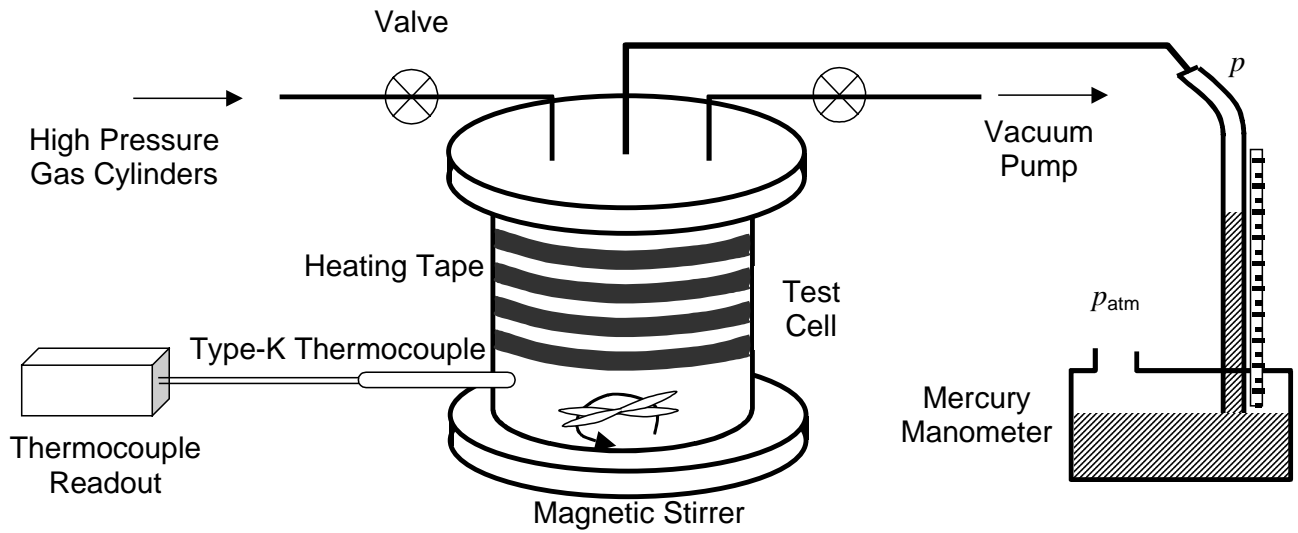


Figure 1. Schematic of the experimental apparatus.

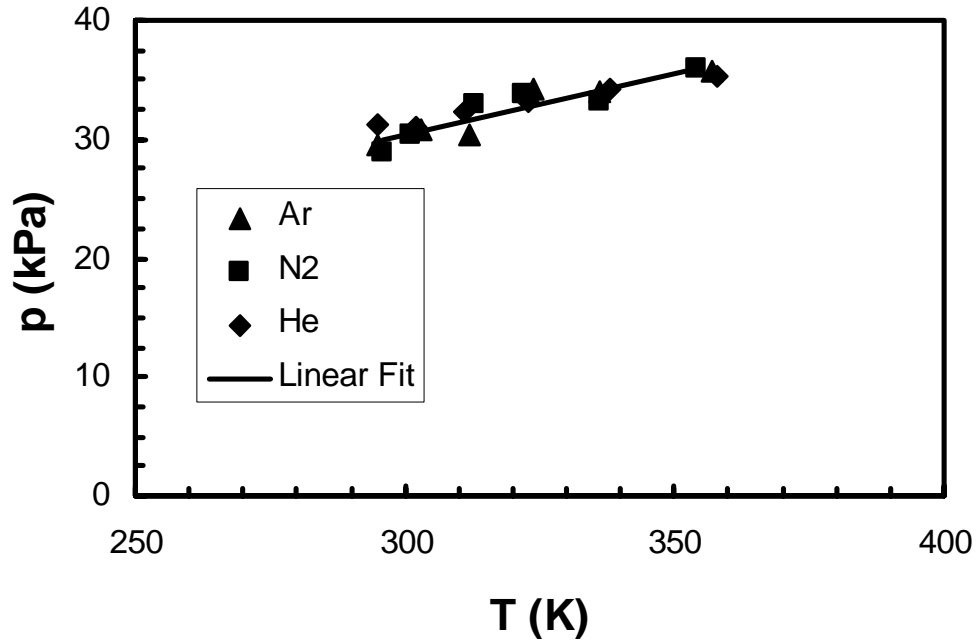


Figure 2. Absolute pressure dependence on absolute temperature for three fixed density gases. In addition to the data points, the graph also shows a linear data fit that passes through the origin ( $T=0, p=0$ ).

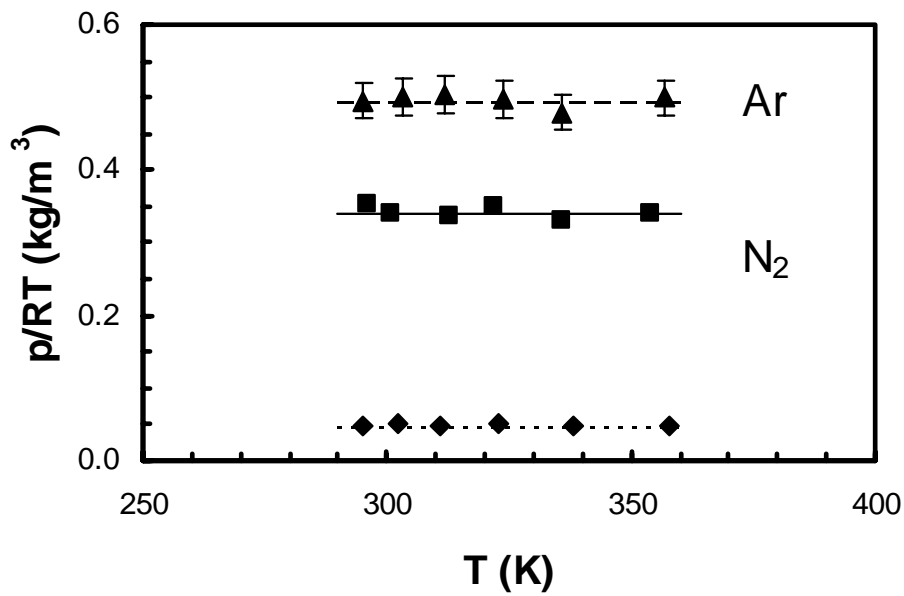


Figure 3. Variation in the normalized pressure/temperature ratio for three gases of constant mass in a fixed volume container. Also shown are the average values for each data set, and the uncertainty in the Ar density based on the analysis in the Error Analysis section.

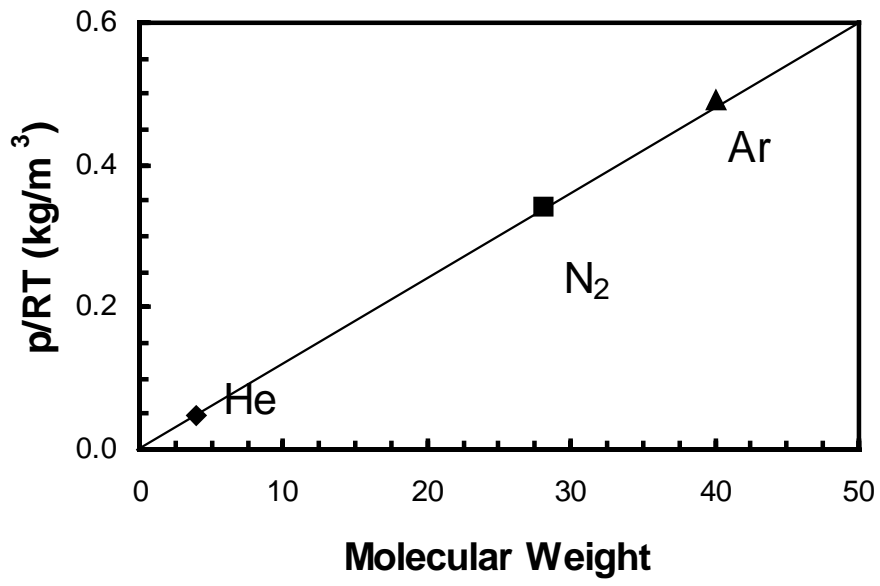


Figure 4. Dependence of the density for the three gases as a function of molecular weight of the test gas.