Uncertainty in Measurements

References

- AIAA S-071A-1999 *Assessment of Experimental Uncertainty With Application to Wind Tunnel Testing*, 1999.
- NIST Technical Note 1297 *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, 1994.

DEFINITIONS

Statistic	Equation
Average or Mean of the Sample	$\overline{y} = \left(\sum_{1}^{N} y_i \right) / N$
Sample Standard Deviation	$S_{N-1} = \sqrt{\frac{1}{N-1} \sum (x_i - \overline{x})^2}$
Population Standard Deviation	$\sigma_x = \sqrt{\frac{1}{N} \sum (x_i - \overline{x})^2}$
Standard Deviation or Standard Error of the Mean or Average	$\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{N}}$
Note: All based on Gaussian (Normal) Distribution	

Estimates for N measurements of a normally distributed quantity X

Topic	Value
Best Estimate for X the "true value"	$\bar{X}$
Best Estimate for width of measurement distribution	σ_X
Estimate for uncertainty in $\bar{X}$ (67/68% confidence level)	$\sigma_{\bar{X}}$
Estimate for uncertainty in $\bar{X}$ (95% confidence level)	$1.96\ \sigma_{\bar{X}}$

Estimating Uncertainties in Calculated Quantities –Using Partial Derivatives

- Assume calculated quantity is of the form $G=f(y_1,y_2,y_3,\ldots)$
- σ is the uncertainty on each quantity, y
- The estimated uncertainty in G is given by

$$\sigma_G^{\;2} = \sum_i \left[\frac{\partial G}{\partial y_i} \sigma_{y_i} \right]^2$$

Sample Calculation - measurement of the velocity, V, of an air stream inside a duct using a pitot-static tube

$$V = \sqrt{\frac{2(\Delta p)RT_a}{p_a}} \qquad \sigma_C = ?$$

100 measurements of Δp - mean = 8.0 inH2O, rms = 0.5 inH2O
0.15% FS accuracy (FS is 10 inH2O)
Single reading of T – 527.1 R, 0.2 degR accuracy
Single reading of Pa – 14.7 psia, 2% full scale accuracy (15psia sensor)

Pa = 14.7 +/- 0.3 psia (bias error)
T = 527.1 +/- 0.2 R (bias error)

$$\sigma_p = \sqrt{\left[\sigma_{bias}{}^2 + \sigma_{precision}{}^2\right]}$$

$$\sigma_p = \sqrt{\left[(0.0015*10)^2 + \frac{1.96*prms}{\sqrt{100}}\right]}$$

$$\sigma_p = 0.1$$

Sample Calculation - measurement of the velocity, V, of an air stream inside a duct using a pitot-static tube

$$\mathbf{V} = \sqrt{\frac{2(\Delta p)RT_a}{p_a}}$$

Δp = 8.0 ± 0.1 in. H2O (confidence level 95 %)

Ta = 527.1 ± 0.2°R

pa = 14.7 ± 0.3 psia

$$\sigma_C = \left\{ \frac{1}{2}\frac{RT_a}{(\Delta p)p_a}\left(\sigma_{\Delta P}\right)^2 + \frac{1}{2}\frac{(\Delta p)RT_a}{p_a^3}\left(\sigma_{Pa}\right)^2 + \frac{1}{2}\frac{R(\Delta p)}{T_a p_a}\left(\sigma_{Ta}\right)^2 \right\}^{1/2}$$

$$\frac{\sigma_C}{\mathbf{V}} = ?$$

Sample Calculation - measurement of the velocity, V, of an air stream inside a duct using a pitot-static tube

$$V = \sqrt{\frac{2(\Delta p)RT_a}{p_a}}$$

Δp = 8.0 ± 0.1 in. H2O (confidence level 95 %)

Ta = 527.1 ± 0.2°R

pa = 14.7 ± 0.3 psia

$$\sigma_C = \left\{ \frac{1}{2}\frac{RT_a}{(\Delta p)p_a}(\sigma_{\Delta P})^2 + \frac{1}{2}\frac{(\Delta p)RT_a}{p_a^3}(\sigma_{Pa})^2 + \frac{1}{2}\frac{R(\Delta p)}{T_a p_a}(\sigma_{Ta})^2 \right\}^{1/2}$$

$$\frac{\sigma_C}{V} = \left\{ \left[\frac{1}{2}\frac{\sigma_{\Delta P}}{\Delta p}\right]^2 + \left[\frac{1}{2}\frac{\sigma_{Pa}}{p_a}\right]^2 + \left[\frac{1}{2}\frac{\sigma_{Ta}}{T_a}\right]^2 \right\}^{1/2}$$

Sample Calculation - measurement of the velocity, c, of an air stream inside a duct using a pitot-static tube

$$\mathbf{V} = \sqrt{\frac{2(\Delta p)RT_a}{p_a}}$$

Δp = 8.0 ± 0.1 in. H2O (confidence level 95 %)

Ta = 527.1 ± 0.2°R

pa = 14.7 ± 0.3 psia

$$\sigma_C = \left\{ \frac{1}{2}\frac{RT_a}{(\Delta p)p_a}(\sigma_{\Delta P})^2 + \frac{1}{2}\frac{(\Delta p)RT_a}{p_a^3}(\sigma_{Pa})^2 + \frac{1}{2}\frac{R(\Delta p)}{T_a p_a}(\sigma_{Ta})^2 \right\}^{1/2}$$

$$\frac{\sigma_C}{\mathbf{V}} = \left\{ \left[\frac{1}{2}\frac{\sigma_{\Delta P}}{\Delta p}\right]^2 + \left[\frac{1}{2}\frac{\sigma_{Pa}}{p_a}\right]^2 + \left[\frac{1}{2}\frac{\sigma_{Ta}}{T_a}\right]^2 \right\}^{1/2}$$

$$\frac{\sigma_C}{\mathbf{V}} = \frac{1}{2}\left[1.56x10^{-4} + 4.16x10^{-4} + 1.44x10^{-7}\right]^{1/2} = 0.01197 \; or \; 1.2\%$$