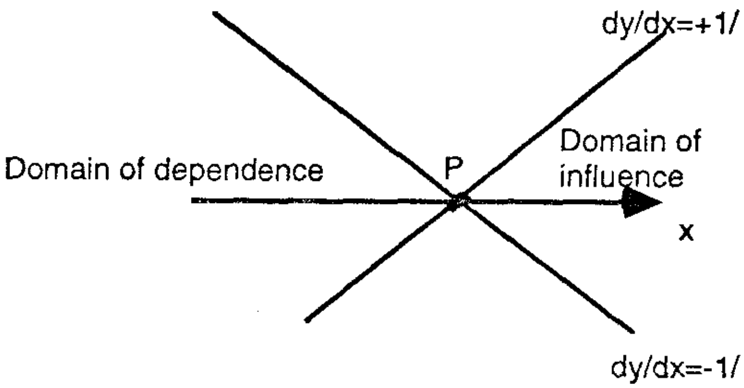
Example: Classify the TSD equation

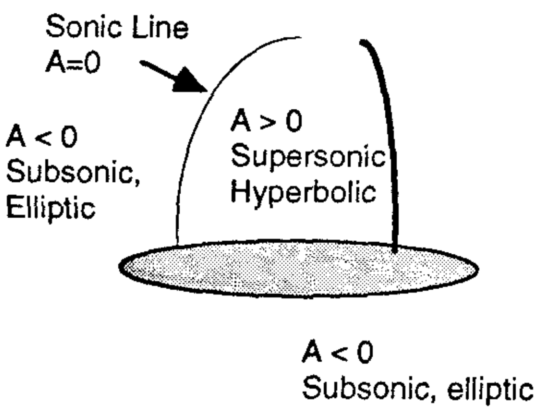
$$\underbrace{\left[1-M_\infty^2-(\gamma+1)M_\infty^2\left(\frac{\phi_x'}{V_\infty}\right)\right]}_{\equiv A}\phi_{xx}'+\phi_{yy}'=0$$

At any point P in space, these two characteristics will have slopes that are equal in magnitude and opposite in sign, and will be symmetric about the x-axis, as shown. The region in front of the point P enclosed by the characteristics is known as the <u>domain of dependence</u> of point P. The region downstream of P, enclosed within the characteristics is influenced by point P and known as the <u>domain of influence</u> of P.



In a general transonic flow, the quantity A can change sign from point to point. Thus, the TSD equation may be elliptic in some (subsonic) regions of the flow, parabolic on sonic lines, and supersonic and hyperbolic in other regions:

TSD Math Classification



Any numerical scheme must account for the fact that these three regions may simultaneously exist in a transonic flow. The numerical scheme must be

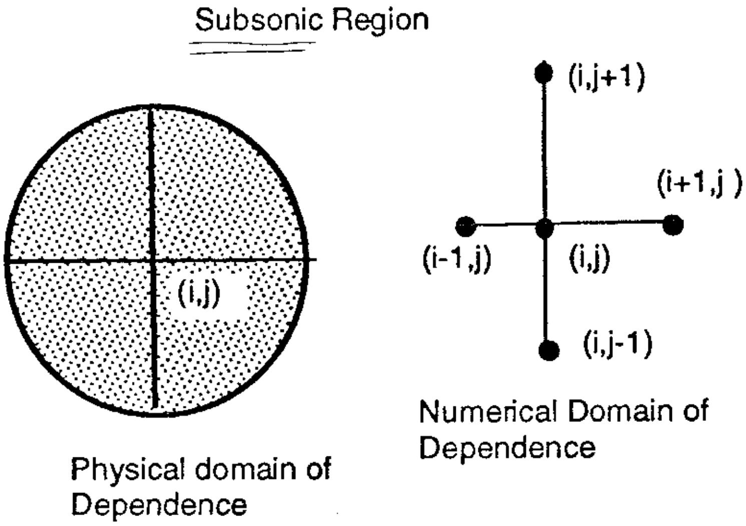


Figure 4.6 In elliptic regions, the node (i,j) depends on the surrounding region

In supersonic regions, however, the point P is not influenced by points outside the "Mach" cone, a wedge formed by the two characteristic lines, discussed in Chapter III.

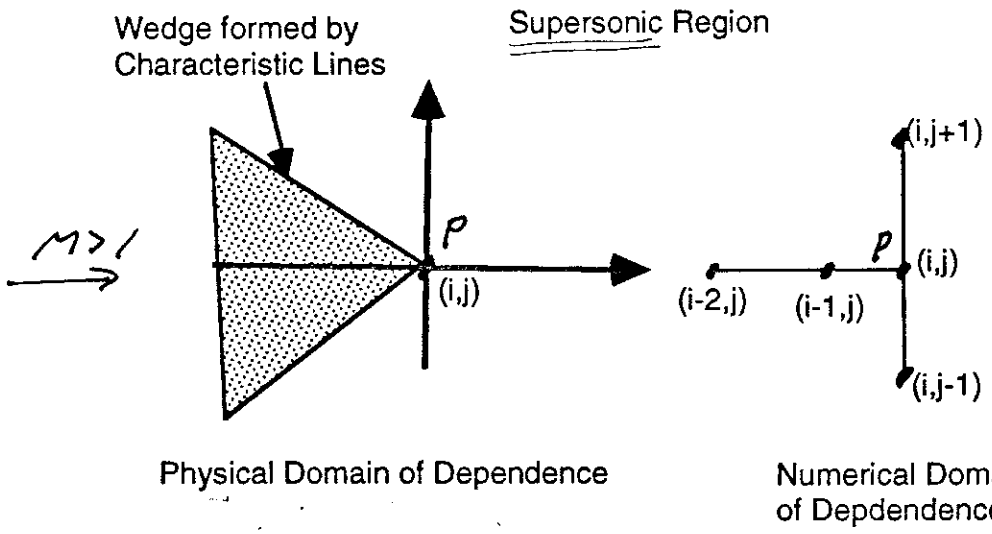


Figure 4.7 In supersonic regions, the node (i,j) should depend primarily on the information within the characteristic cone

Murrman and Cole argued that in supersonic regions, the finite difference approximation should reflect the upstream bias, required by the physics of the flow.

• Because in supersonic flow in positive x direction disturbances propagate from the left, Murman + Cole suggested in supersonic flow use:

$$\underbrace{\frac{P_{i-1/2,j} - P_{i-3/2,j}}{\Delta X_i + \Delta X_{i+1}}} + \frac{Q_{i,j+1/2} - Q_{i,j-1/2}}{\Delta Y_j + \Delta Y_{j+1}} = 0$$

Has been shifted to left to follow domain of dependence.

• The mathematical classification depends on A (as shown earlier). Thus this scheme yields:

$$\boxed{\begin{gathered} \mu_{i,j} = \begin{cases} 1 \text{ if } A_{i,j} < 0 \\ 0 \text{ if } A_{i,j} > 0 \end{cases} \\[2em] \frac{P_{i+1/2,j} - P_{i-1/2,j}}{\Delta X_i + \Delta X_{i+1}} + \frac{Q_{i,j+1/2} - Q_{i,j-1/2}}{\Delta Y_{j+1} + \Delta Y_j} \\ + \mu_{i,j}\left[\frac{P_{i-1/2,j} - P_{i-3/2,j}}{\Delta X_i + \Delta X_{i+1}} - \frac{P_{i+1/2,j} - P_{i-1/2,j}}{\Delta X_i + \Delta X_{i+1}}\right] = 0 \end{gathered}}$$

Murman + Cole (1971 Scheme)