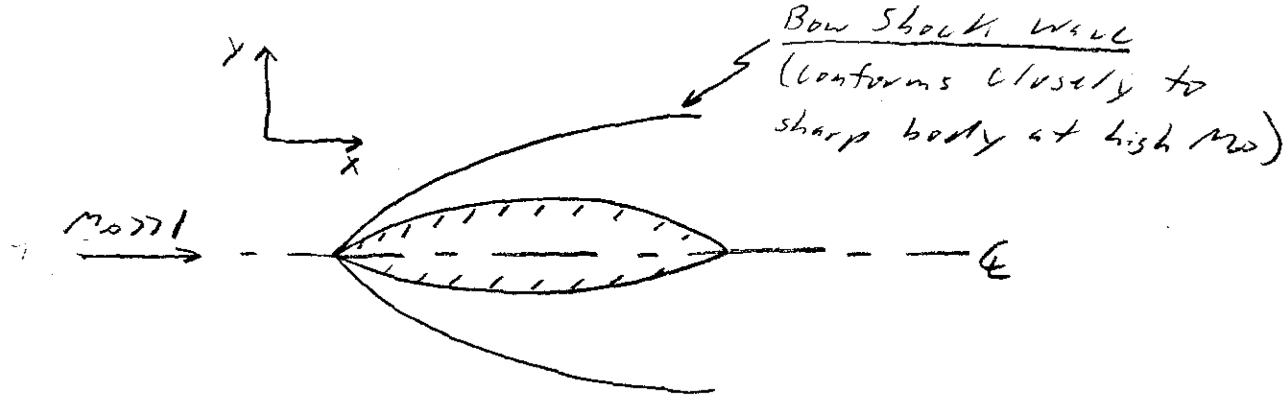
Other Surface Inclination Methods

* Modified Newtonian method works well for blunt bodies but for sharp-nosed bodies Modified Newtonian does not perform well.
* We will now summarize several surface inclination methods designed for sharp-nosed bodies.

Tangent Wedge Method

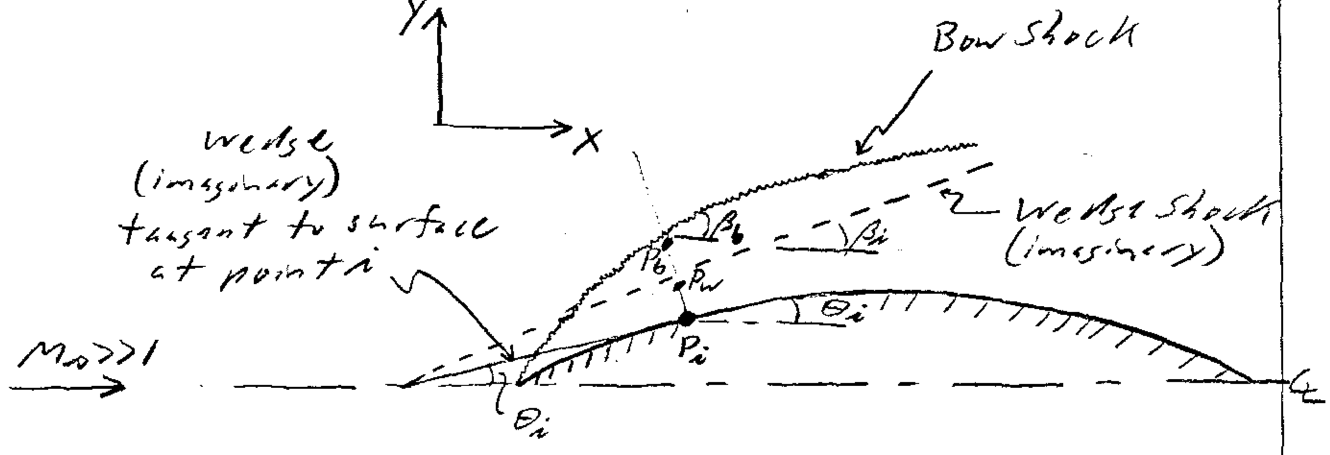
* This <u>approximate</u> method is valid for 2-D, hypersonic flows with <u>attached</u> shock waves (ie. sharp noses and high M_∞)

- Consider, for example, flow around a biconvex airfoil



- To find the <u>surface pressure</u> we approximate the body by a series of <u>wedges</u>.

- For simplicity, consider the upper half of the body:



- To find P_i (surface pressure at i):

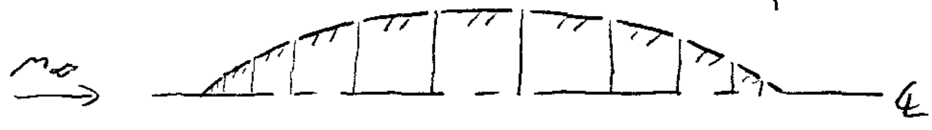
1) Determine body shape at point i:

$$\theta_i = \tan^{-1}\left(\frac{dy}{dx}\right)_i$$

2) Find the oblique shock angle β_i which corresponds to wedge angle θ_i and M_0. Find P_w (post shock pressure) for this shock

3) Approximate $\underline{P_i \approx P_w}$ as the surface pressure

- The entire body can be computed by dividing it up into piecewise linear segments. Apply the procedure above to each successive segment



- Why does this approximation work?
  - In hypersonic flow around a sharp body, the shock is close to the body and has a dominant effect on surface pressure
  - For most such bodies we expect:
    - o P_i somewhat < P_b (due to expansion upstream of i)
    - o P_w somewhat < P_b (wedge shock angle < bow shock angle)

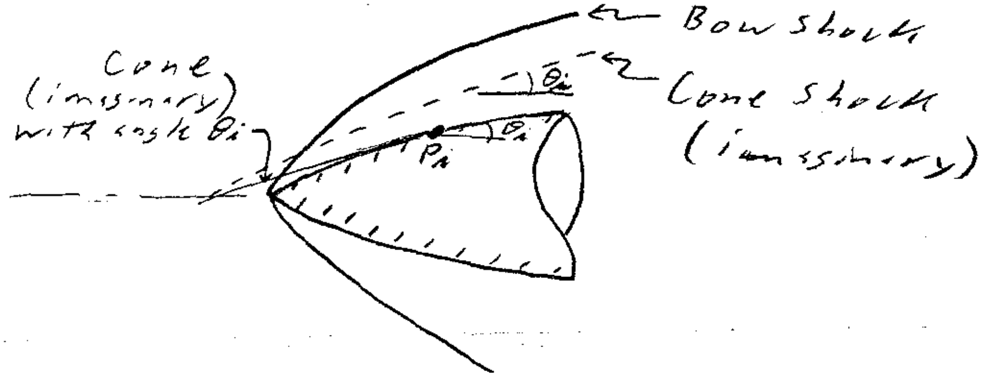
  Thus this method approximates:

    - o $P_i \approx P_w$ somewhat < P_b

  - This method fails if any of the wedge angles corresponds to a detached shock wave

Tangent Cone Method

* This approximate method is valid for 3-D, hypersonic flows with attached shock waves (i.e. sharp noses and high M_∞)



• Applies the same approach as Tangent Wedge Method but uses oblique cone tables (3-D) instead of oblique shock tables (2-D)

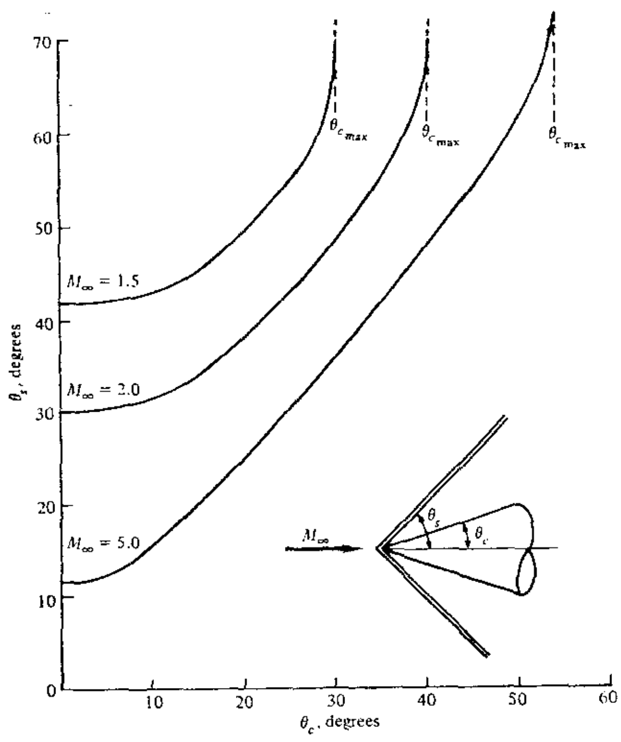
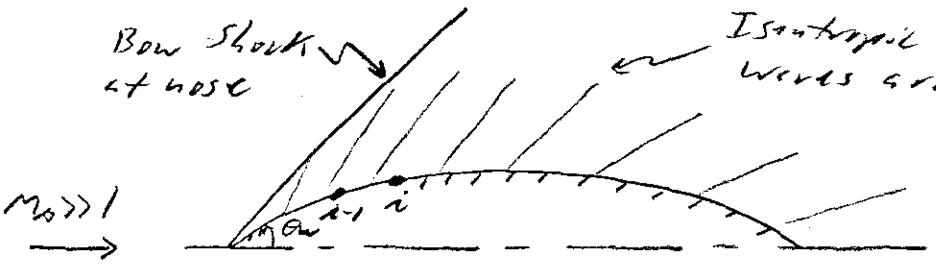


FIGURE 10.5
θ_c-θ_s-M diagram for cones in supersonic flow. (The top portion of the curves curl back for the strong shock solution, which is not shown here.)

Shock - Expansion Theory

* This approximate method is valid for 2-D, hypersonic flows with <u>attached</u> shock waves (i.e. sharp noses and high M_∞)



- To find P_i (surface pressure at i)

  1) Use oblique shock relations for M_∞ and nose angle θ_w to find properties at the nose: P_0, P, M, and ν (Prandtl-Meyer angle)

  2) Use Prandtl-Meyer relations to find ν_i (and thus M_i) in expansions around the remainder of the body:

$$\underline{\nu_i = \nu_{i-1} + (\theta_{i-1} - \theta_i)}$$

  3) Determine P_i from known ν_i, M_i and $P_{0w} = P_{0i-1} = P_{0i}$ and from isentropic relations.

Note:

- This method ignores interactions of the expansion waves with the shock wave.
- This method tends to work better in high M_∞ flow rather than just supersonic flow

Supersonic Flow
μ large

Hypersonic Flow
μ small

$\mu = \sin^{-1}\left(\frac{1}{M}\right) \equiv$ Mach Angle

- In hypersonic flow, the characteristic reflections off the shock are much less likely to interact with the body and affect P_i.