AE6020 | HW #1

Solution

1. - from figure 161,

- at $M_\infty = 0.9$ and $\alpha = 2°$, $\underline{C_l = 0.14}$
- at $M_\infty = 0.4$ and $\alpha = 2°$, $\underline{C_l = 0.15}$

- Prandtl - Glauert Rule;

$$C_l = \frac{C_{l_0}}{\sqrt{1-M_\infty^2}}$$

$$C_{l_0} = C_l \sqrt{1-M_\infty^2}$$

- using point at $M_\infty = 0.4, \alpha = 2$;

$$C_{l_0} = 0.15 \sqrt{1 - 0.4^2}$$

$$\underline{= 0.14}$$

- prediction for $M_\infty = 0.9, \alpha = 2$

$$C_l = \frac{C_{l_0}}{\sqrt{1-M_\infty^2}} = \frac{0.14}{\sqrt{1-0.9^2}} \underline{= 0.32}$$

∴ predicted $C_l = 0.32$

actual $C_l = 0.14$

$$\% \text{ error} = \frac{|0.32 - 0.14|}{0.14} \times 100\%$$

$$\boxed{= 128\%}$$

2) Since the flow is described as very low-speed, I will assume that the incompressible assumption is valid. Therefore $C_{p_0} = -0.65$ To find the pressure coefficient at $M_\infty = 0.7$ we can apply different compressibility corrections.

a) Prandtl - Glauret $\quad C_p = \dfrac{C_{p_0}}{\sqrt{1-M_\infty^2}} = \dfrac{-0.65}{\sqrt{1-.49}} = \boxed{-0.91}$

b) Karmen - Tsien

$$C_p = \frac{C_{p_0}}{\sqrt{1-M_\infty^2} + \left(\frac{M_\infty^2}{1+\sqrt{1-M_\infty^2}}\right)\frac{C_{p_0}}{2}} = \frac{-0.65}{\sqrt{1-.49} + \left(\frac{0.49}{1+\sqrt{1-.49}}\right)\left(\frac{-.65}{2}\right)}$$

$$= \frac{-0.65}{0.714 - 0.0929} = \boxed{-1.05}$$

c) Laitone (assume $\gamma = 1.4$)

$$C_p = \frac{C_{p_0}}{\sqrt{1-M_\infty^2} + \left(\frac{M_\infty^2\left(1+\frac{\gamma-1}{2}M_\infty^2\right)}{2\sqrt{1-M_\infty^2}}\right)C_{p_0}} = \frac{-0.65}{\sqrt{1-.49} + \left(\frac{.49(1+.2(.49))}{2\sqrt{1-.49}}\right)(-.65)}$$

$$= \frac{-0.65}{0.714 - 0.245} = \boxed{-1.39}$$

3) If we assume that the flowfield is isentropic, we can follow the derivation in Modern Compressible Flow (Section 9.7) to get the following expression for critical Mach number:

$$C_{p,cR} = \frac{2}{\gamma M_{cr}^2}\left[\left(\frac{1+\left(\frac{\gamma-1}{2}\right)M_{cr}^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

Once again we assume $\gamma = 1.4$ (air @ STP conditions)
Using the given value of $C_{p_0} = -0.41$, we set the above expression equal to the Prandtl-Glauret rule to find the critical mach number

$$\frac{C_{p_0}}{\sqrt{1-M_{cR}^2}} = C_{p,cR} = \frac{2}{\gamma M_{cr}^2}\left[\left(\frac{\left(1+\frac{\gamma-1}{2}M_{cr}^2\right)}{\left(1+\frac{\gamma-1}{2}\right)}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

$$-0.41\left(\frac{1.4}{2}\right) = \frac{\sqrt{1-M_{cr}^2}}{M_{cr}^2}\left[\left(\frac{1+0.2M_{cr}^2}{1.2}\right)^{\frac{7}{2}} - 1\right]$$

Solving in Mathematica, I obtained: $\boxed{M_{cR} = 0.74}$

(Note to solve by hand an iterative method such as Newton's method would be applied.)

This solution could also be found graphical by plotting the two forms of $C_{p,cR}$ above and observing where they intersect.

4a) Derived in section 9.7 of the text, the critical pressure coefficient is given by the following formula:

$$C_{p,cr} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{1+\frac{\gamma-1}{2}M_\infty^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

To apply this relation to the problem of interest two assumptions will need to be made:

1) The flow is isentropic. This assumption is necessary to derive the above expression

2) $\gamma = 1.4$ This information is not given directly so we will assume the value for air at STP

$$C_{p,cR} = \frac{2}{(1.4)(0.87)^2}\left[\left(\frac{1+(0.2)(0.87)^2}{1+0.2}\right)^{3.5} - 1\right] = \boxed{-0.25}$$

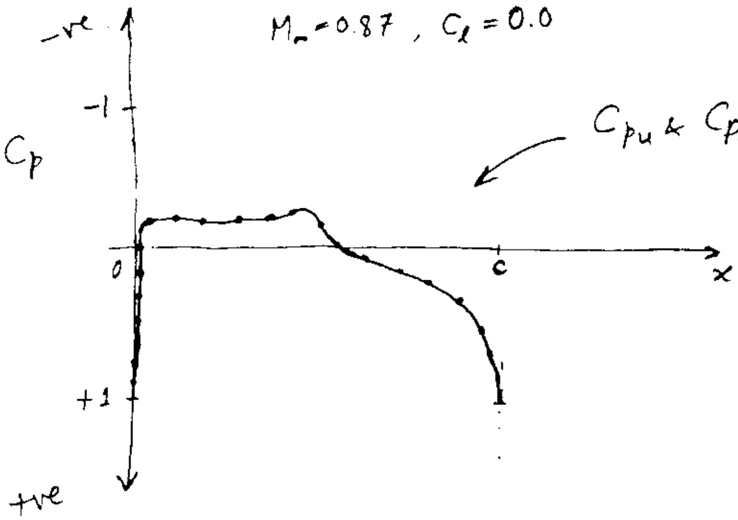
This value represents the pressure coefficient at the point on the airfoil where the flow is sonic. This value agrees with the marked value of $C_{p,cR}$ shown in figure 14.1

b) The critical pressure coefficient (for each M_∞) is shown as a dashed line on the plots. In the figures if the C_p distribution never exceeds this $C_{p_{CR}}$ then the flow is subcritical.

c) By definition, lift is found by integrating the pressure about the airfoil. With a basic knowledge of calculus we know that integration also gives the area under a curve. Therefore we can tell if the airfoil is generating lift by the area between the upper and lower surface pressure coefficient distributions. If convention is followed and the negative of C_p is plotted then positive lift would occur if the majority of the upper surface distribution was above that of the lower. In this case the upper surface C_p's are greater than the lower surface C_p's indicating that this airfoil is generating lift. (Note the negative of C_p is actually plotted here which means the upper surface pressure is lower than the bottom surface.)

Since NACA 64A009
is symmetric

(d)



5. Observe that -

1. Lift curve slope i.e $\frac{dC_l}{d\alpha}$ is lower in the supercritical case.
2. Cd is significantly higher in the supercritical regime
3. Cl/Cd remains low with very little growth with increasing α for supercritical case.

The occurence of shocks in the supercritical regime is responsible for the differences.

→ Shock-induced separation causes loss of high suction on the upper surface, hence loss of lift.

→ Shock-induced separation causes higher pressure drag. This combined with wave drag causes high drag rise.

→ Loss in lift & higher drag together contribute to low Cl/Cd.

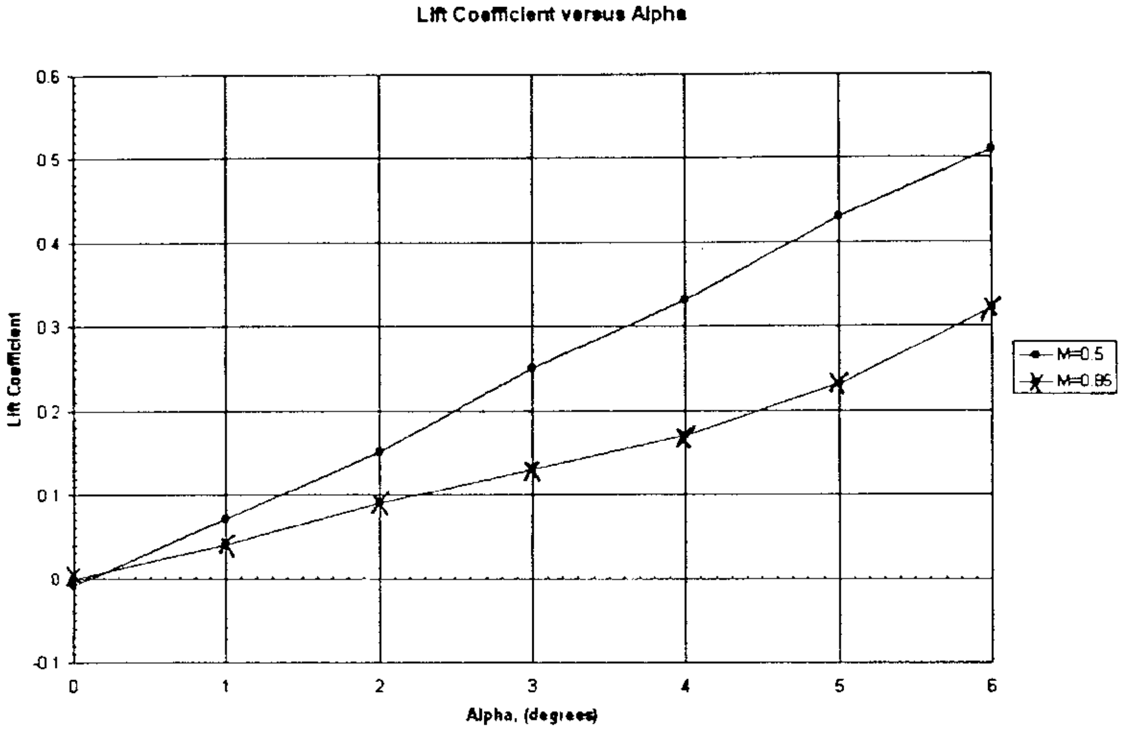


Figure 1. Lift coefficient as a function of alpha for two mach numbers (NACA 0012-34)

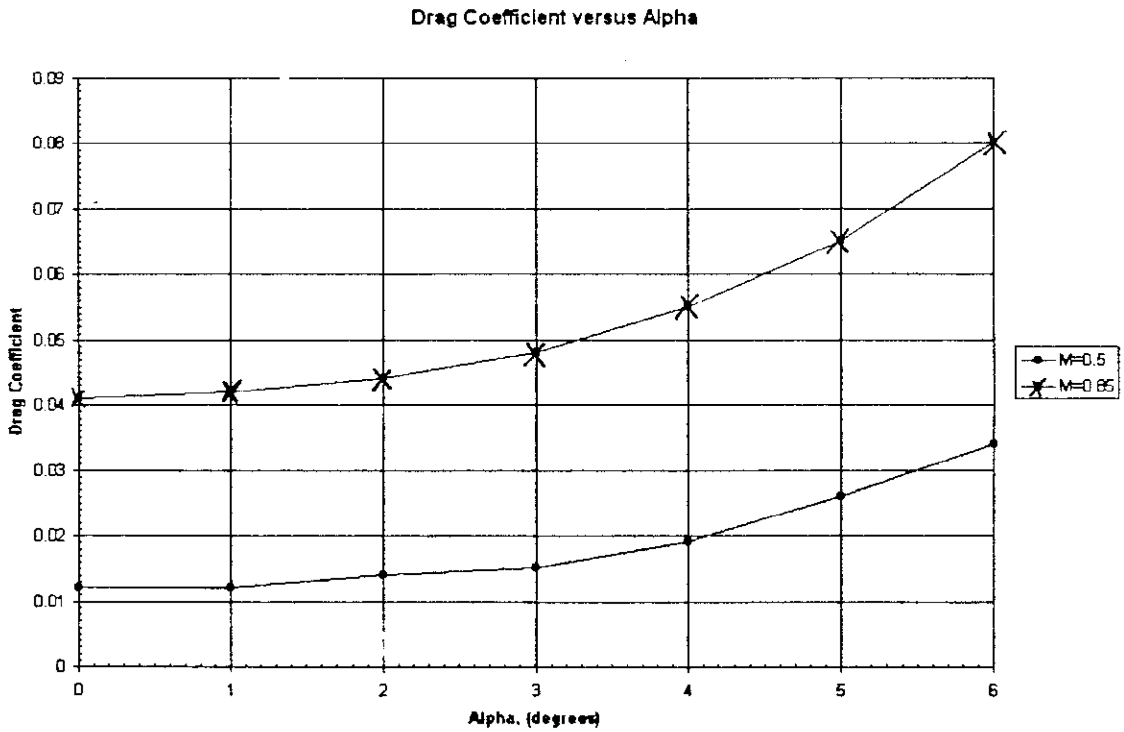


Figure 2. Drag coefficient as a function of alpha for two mach numbers (NACA 0012-34)

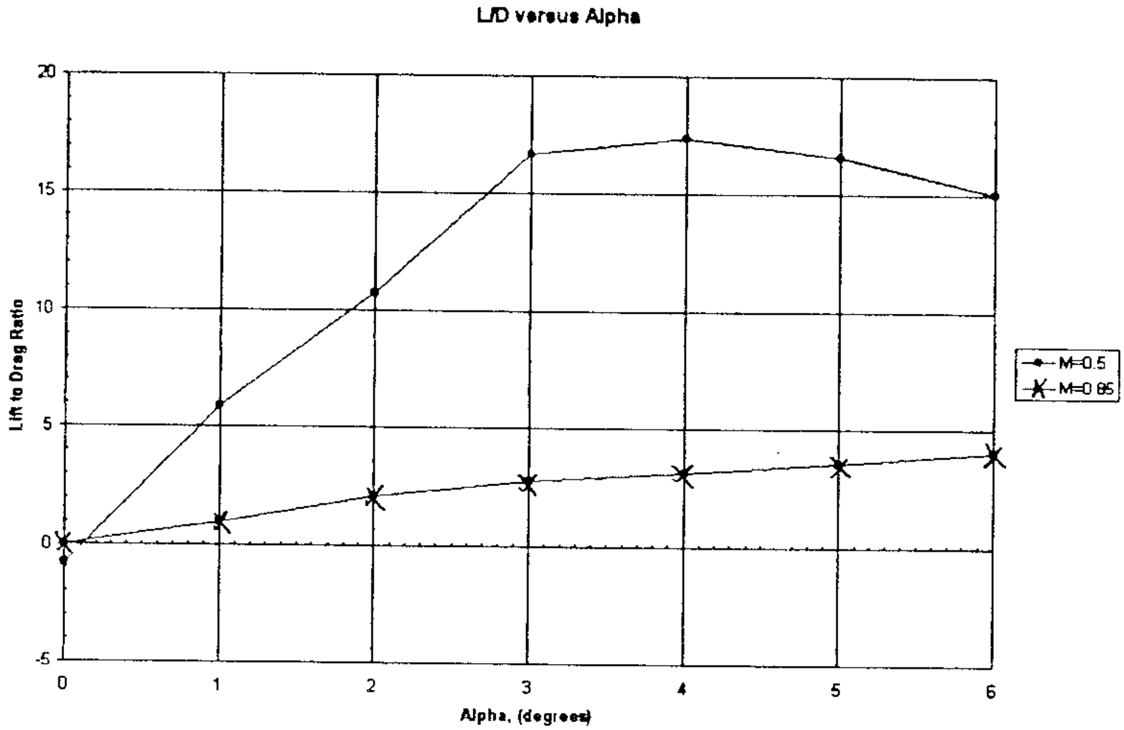


Figure 3. Lift to drag ratio as a function of alpha for two mach numbers (NACA 0012-34)