

**AE 6020 High-Speed Flow  
Assignment #4**

1) The steady conservation equations across a strong normal shock wave (i.e. hypersonic speed) are given by:

$$\begin{array}{ll} \text{mass} & \rho_1 u_1 = \rho_2 u_2 \\ \text{momentum} & \rho_1 u_1^2 \approx p_2 + \rho_2 u_2^2 \\ \text{energy} & \frac{1}{2} u_1^2 \approx h_2 + \frac{1}{2} u_2^2 \end{array}$$

a) First, write out the exact normal shock equations for momentum and energy and then explain why the approximate equations shown above are valid when the flow upstream of the shock is hypersonic.

b) If the gas is approximated as thermally and calorically perfect behind the shock then

$$h_2 = \frac{\gamma_2}{\gamma_2 - 1} \frac{p_2}{\rho_2}$$

Without assuming that the ratio of specific heats is the same on each side of the shock, show that for strong shocks:

$$\varepsilon \equiv \frac{\rho_1}{\rho_2} \approx \frac{\gamma_2 - 1}{\gamma_2 + 1}$$

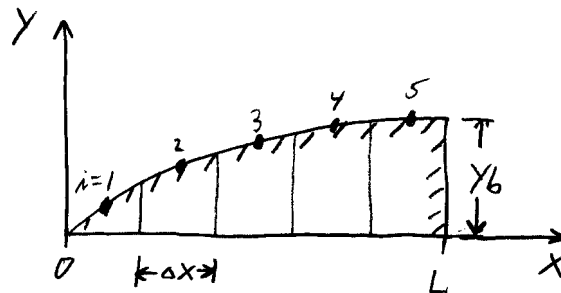
where  $\gamma_2$  is the effective value of the ratio of specific heats behind the shock wave.

2) The curves shown in Figure 15.9 in the Anderson text are changed when skin friction on the plate is included. In particular, the variation of  $L/D$  with  $\alpha$  will peak at a low angle of attack, and go to zero at  $\alpha=0$ . In this problem we investigate why this phenomenon. Let the drag coefficient due to skin friction be assumed constant and denoted by  $C_{D_0}$ .

a) Assuming a Newtonian pressure distribution, show that the maximum value of  $L/D$  is  $0.667/(C_{D_0})^{1/3}$ , and occurs at an angle of attack (in radians) of  $\alpha = (C_{D_0})^{1/3}$ .

b) Furthermore, at  $(L/D)_{\max}$ , show that  $C_{D_0} = \frac{1}{3} C_D$  where  $C_D$  is the total drag coefficient. (In other words, we can state that at  $(L/D)_{\max}$ , wave drag is twice the friction drag)

3) Consider the 2-D body shown whose shape is described by  $y = C_1 x + C_2 x^2$ .



In this problem you will determine the sectional (i.e. 2-D) drag coefficient based on length  $L$  using five approximate methods applicable in hypersonic flow. To apply these methods divide

the body into 5 linear panels with equal  $\Delta x$  on each panel. Use the following numerical values in your analysis:

$$M_\infty = 8.0 \quad C_1 = 0.5774$$

$$\gamma = 1.4 \quad C_2 = -0.2887$$

$$L = 1.0$$

For each method give  $c_d$  and the % difference relative to the Shock Expansion result.

- a) Newtonian
- b) Modified Newtonian
- c) Tangent/Wedge using exact shock relations to find pressure on each panel
- d) Tangent/Wedge using equations 15.11 and 15.15 to approximate pressure coefficient on each panel
- e) Shock Expansion

Neatly summarize your results in a table.