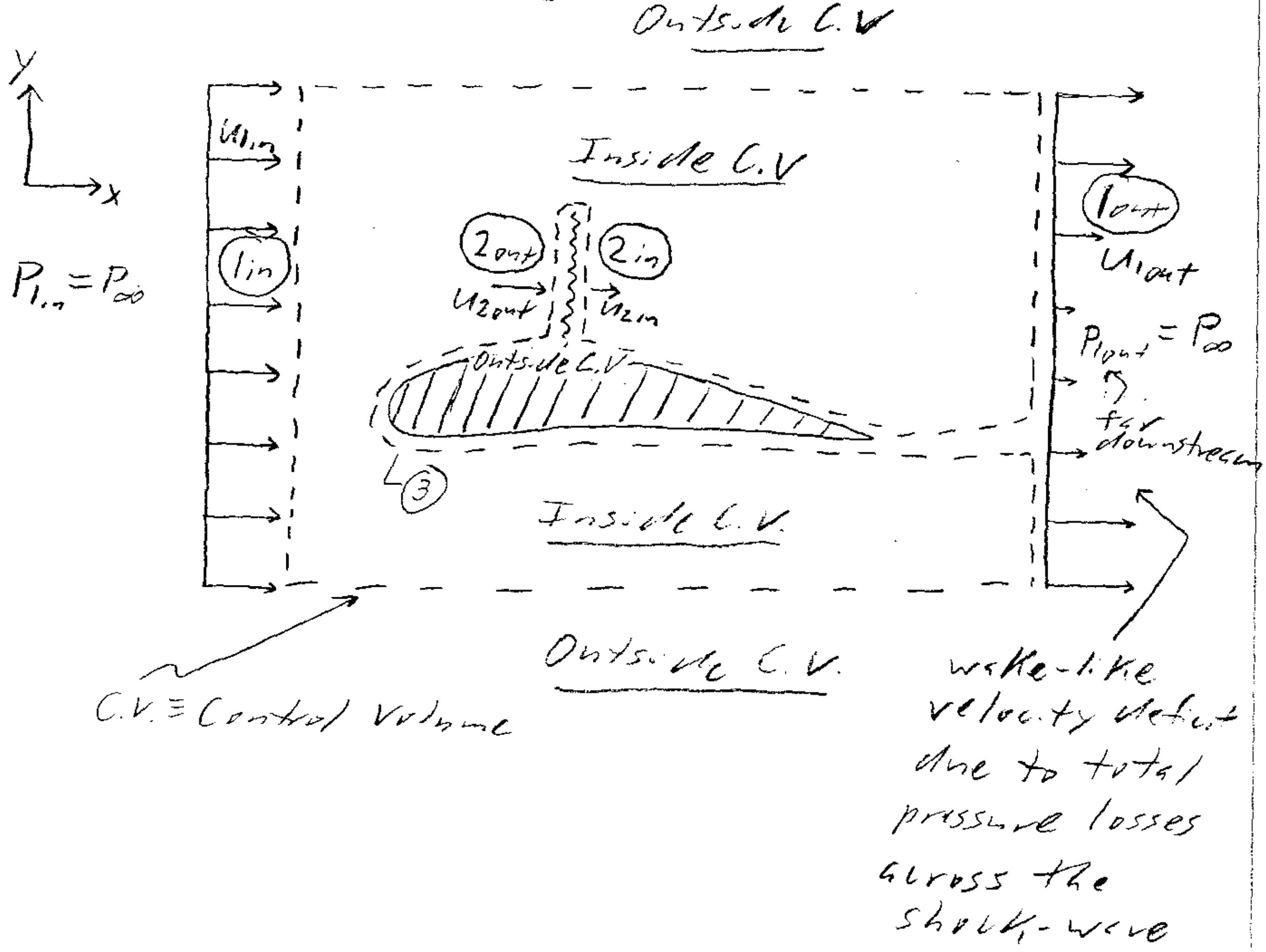
Q: Since the FPE model of a shock wave artificially increases the momentum of the flow (eg. $P+\rho u^2$ increased from 1.496 to 1.500 on page 2-11), why doesn't this increase in momentum lead to thrust and not drag?

A: First recognize that the FPE satisfies the inviscid momentum equation except through the shock wave (since it is non-isentropic).

Thus, let's apply the integral momentum equation to a control volume that does not go through the shock wave...

Euler Eqns



Main Inflow ≡ (1in) (Flow moves <u>into</u> C.V.)

Main Outflow ≡ (1out) (Flow moves <u>out</u> of C.V.)

Shock Downstream ≡ (2in) (Flow moves <u>into</u> C.V.)

Shock Upstream ≡ (2out) (Flow moves out of C.V.)

Airfoil ≡ (3) (Drag felt by airfoil

Euler Eqns (continued)

- Let's now apply the inviscid, integral form of the momentum equation in the x-direction (See Low-Speed Eqn Summary).

Note: The signs are easier to handle because at ① & ② $u>0$...

$$D_3 = \left(PA + \int \rho u^2 dA\right)_{1in} - \left(PA + \int \rho u^2 dA\right)_{1out}$$
$$+ \left(PA + \int \rho u^2 dA\right)_{2in} - \left(PA + \int \rho u^2 dA\right)_{2out}$$

- Since $P_{1in} = P_{1out}$ this simplifies to:

$$D_3 = \left(\int \rho u^2 dA\right)_{1in} - \left(\int \rho u^2 dA\right)_{1out} + \underbrace{\left[\left(P + \rho u^2\right)_{2in} - \left(P + \rho u^2\right)_{2out}\right] A}_{=0 \text{ for normal shock}}$$

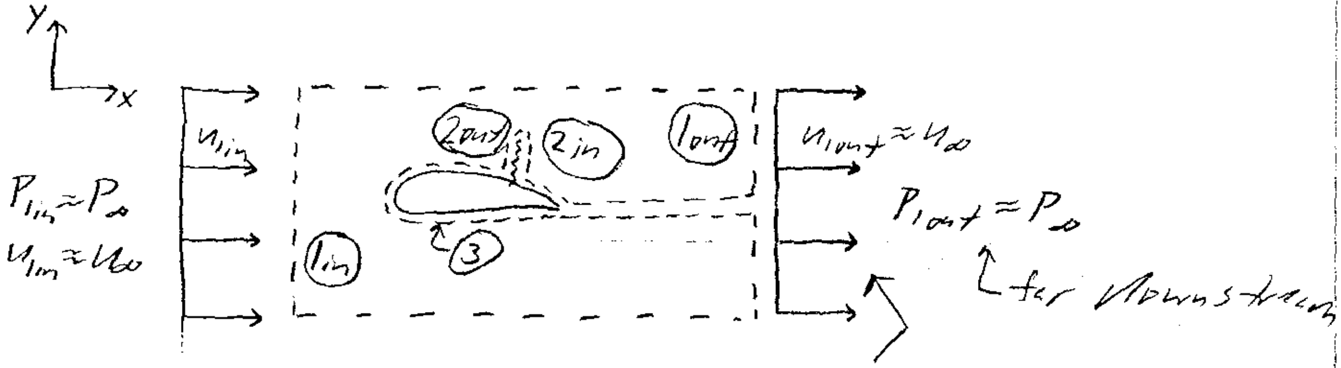
$= 0$ for normal shock, were

$$\boxed{D_3 = \left(\int \rho u^2 dA\right)_{1in} - \left(\int \rho u^2 dA\right)_{1out}}$$

* For Euler Eqns, <u>velocity deficit in wake</u> leads to drag

Due to total pressure losses across the shock.

Full Potential Eqns



velocity out same as velocity in since no total pressure losses

• Again, applying the integral momentum equation in x-direction:

$$D_3 = \underbrace{\left(PA + \int \rho u^2 dA\right)_{1in} - \left(PA + \int \rho u^2 dA\right)_{1out}}_{=0 \text{ since no wake deficit}}$$

$$+ \left(PA + \int \rho u^2 dA\right)_{2in} - \left(PA + \int \rho u^2 dA\right)_{2out}$$

$$D_3 = \left[\underbrace{\left(P + \rho u^2\right)_{2in}}_{\text{Shock Downstream}} - \underbrace{\left(P + \rho u^2\right)_{2out}}_{\text{Shock Upstream}} \right] A$$

Shock Downstream (e.g. 1.500 in class example)

Shock Upstream (e.g. 1.496 in class example)

See page 2

Positive Number thus Drag not Thrust

* For FPE, momentum change across the shock wave leads to airfoil drag