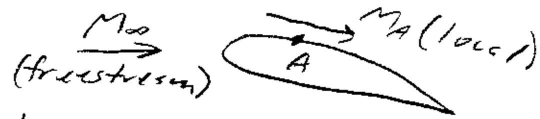
Critical Mach Number

- From the Prandtl-Glauert rule $C_p = \frac{C_{p_0}}{\sqrt{1-M_\infty^2}}$ basically tells us that as M_∞ increases, the <u>local</u> values of M (and P) change:

  Recall isentropic relation:

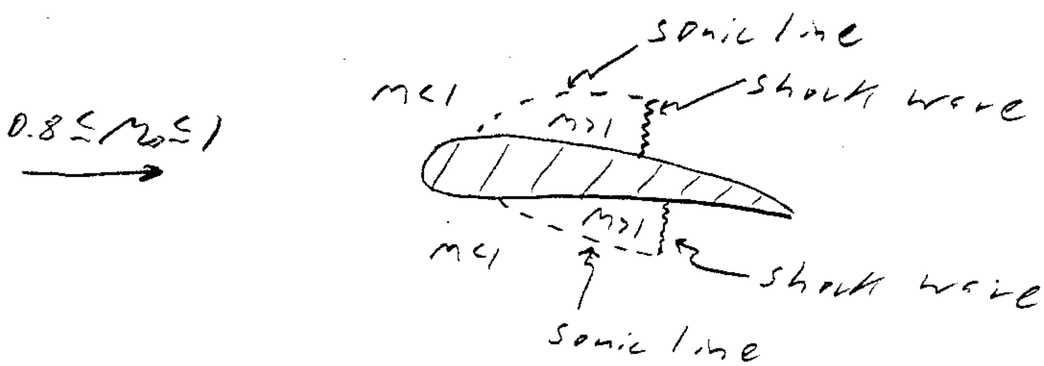
$$P = \frac{P_0 \}\text{= constant in isentropic flow}}{\left[1 + \frac{\gamma-1}{2} M^2\right]^{\gamma/\gamma-1}}$$



  Thus as C_p (or P) $\downarrow$, $M \uparrow$

  local values

- When the freestream velocity is high enough for the <u>local</u> Mach number to be >1, shock waves will begin to form:

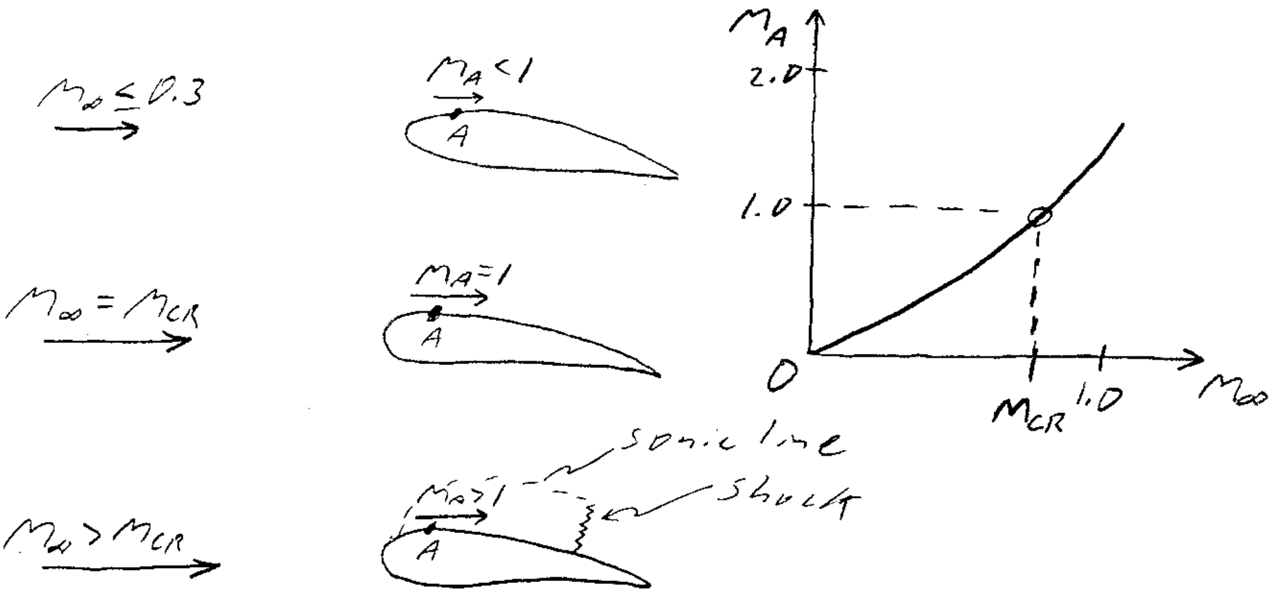


- Recall that when we looked at supersonic flow, we saw that supersonic inviscid flow around a body always yields drag.
  - That <u>pressure drag due to compressibility is wave drag</u>
  - Recall, this wave drag exists even if shock waves are not present.
    (for Supersonic, linearized Potential flow)

- <u>When shock waves exist</u> in the flow, such as in case on previous page,
  <u>wave drag increases significantly</u>.
  - The stronger the shock, the greater the wave drag generated.

Since steady shock waves require that the local flow be supersonic in front of the shock, lets determine when locally supersonic flow will exist on a geometry.

- <u>Let point A be a location on the body where the pressure is a minimum. (ie. suction peak)</u>
- Consider what happens when $M_\infty \uparrow$



$$\boxed{\begin{array}{l}\text{Critical Mach Number} \equiv M_{CR} \\ \equiv \text{Freestream Mach Number } (M_\infty) \text{ at which} \\ \quad \text{sonic flow is first achieved}\end{array}}$$

- <u>Value of M_{CR} is function of α and airfoil shape</u>
- <u>Drag (wave) becomes much larger when $M_\infty > M_{CR}$</u>

• To develop a method to estimate M_{CR}, we first relate P_A to P_∞ realizing that $P_0 = \text{constant}$ (for isentropic)

$$\frac{P_A}{P_\infty} = \frac{(P_0/P_\infty)}{(P_0/P_A)} = \left[\frac{1 + \frac{\gamma-1}{2}M_\infty^2}{1 + \frac{\gamma-1}{2}M_A^2}\right]^{\frac{\gamma}{\gamma-1}}$$

From our earlier expression for C_p:

$$C_{P_A} = \frac{2}{\gamma M_\infty^2}\left(\frac{P_A}{P_\infty} - 1\right) = \frac{2}{\gamma M_\infty^2}\left\{\left[\frac{1 + \frac{\gamma-1}{2}M_\infty^2}{1 + \frac{\gamma-1}{2}M_A^2}\right]^{\frac{\gamma}{\gamma-1}} - 1\right\}$$

• When the critical Mach number is reached:

$$M_\infty = M_{CR}$$

$$M_A = 1.0$$

and $$C_{P_A} = C_{P_{CR}}$$

Critical Pressure Coeff
≡ Local value of C_p when the maximum local Mach number is 1.0.

Plugging these three expressions into the C_p eqn above allows us to relate M_{CR} to $C_{P_{CR}}$:

$$\boxed{C_{P_{CR}} = \frac{2}{\gamma M_{CR}^2}\left\{\left[\frac{1 + \frac{\gamma-1}{2}M_{CR}^2}{1 + \frac{\gamma-1}{2}}\right]^{\frac{\gamma}{\gamma-1}} - 1\right\}} \qquad (6)$$

Note: This expression is not a function of airfoil shape.

• Now, we can estimate M_{cr} using the following procedure:

1) Determine incompressible C_{p_0} at the min. pressure location through either experimental or theoretical methods

2) Plot compressible C_p vs M_∞ based on C_{p_0} and using any of the compressibility corrections

3) Plot $C_{p_{cr}}$ vs M_{cr} from eqn (6). The intersection of the two curves indicates the critical Mach number