Airfoil Design (continued)

2) Inverse Design Techniques

* Pressure distribution is prescribed over the airfoil and airfoil shape is directly computed.
  - Unlike optimization techniques, there is no objective function to be minimized.

• Several inverse design techniques exist. See web notes (Chapter 6) for references. One such method is...

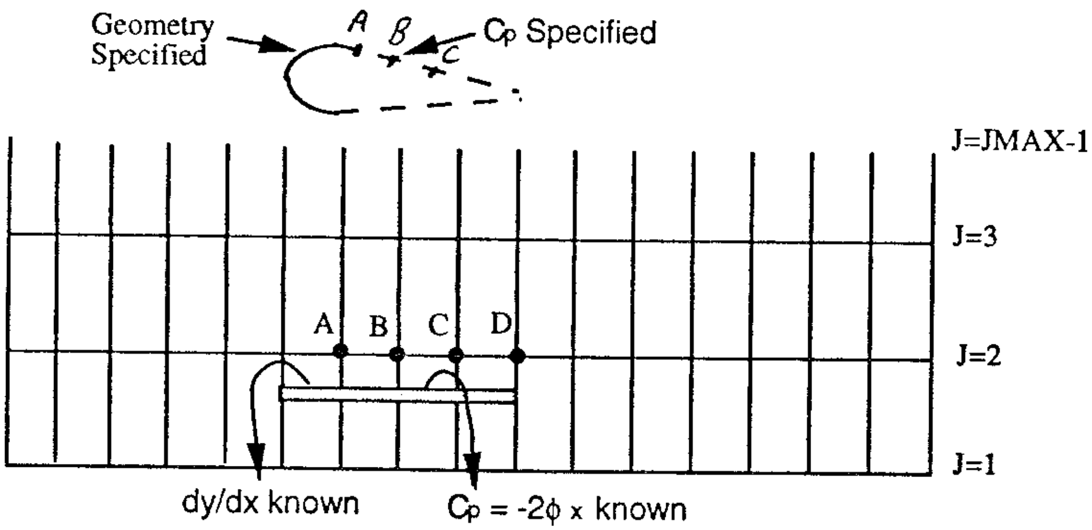
Carlson's Inverse Design Technique

• May be used with TSD on a Cartesian grid.

• May also be used with FPE on a curvilinear grid (body fitted)

• For simplicity, let's consider here application to TSD in a non-lifting flow...

In Carlson's technique, only a part of the airfoil geometry is specified near the nose. Over the rest of the surface, the surface pressure distribution is prescribed.



Using Carlson's approach, we next construct a Cartesian grid and the TSD equation is solved using the principles learnt earlier in this course. At all points, including the points at j=2, away from the wall, the TSD equation is solved as :

*corrections

Murman & Cole

$$(1-\mu_{ij})\frac{P_{i+\frac{1}{2},j}-P_{i-\frac{1}{2},j}}{\Delta x_{i+1}+\Delta x_i}+(\mu_{ij})\frac{P_{i-\frac{1}{2},j}-P_{i-\frac{3}{2},j}}{\Delta x_{i+1}+\Delta x_i}+\frac{Q_{i,j+\frac{1}{2}}-Q_{i,j-\frac{1}{2}}}{\Delta y_{j+1}+\Delta y_j}=0$$

At points adjacent to the airfoil, the above equation is modified depending on whether the point has the body-slope prescribed at i,j-1/2 or c_p prescribed. If the body slope is prescribed, then the discretized equation is

*correction

$$\boxed{(1-\mu_{ij})\frac{P_{i+\frac{1}{2},j}-P_{i-\frac{1}{2},j}}{\Delta x_{i+1}+\Delta x_i}+(\mu_{ij})\frac{P_{i-\frac{1}{2},j}-P_{i-\frac{3}{2},j}}{\Delta x_{i+1}+\Delta x_i}+\frac{Q_{i,j+\frac{1}{2}}-\left.\frac{dy}{dx}\right|_i}{\Delta y_{j+1}+\Delta y_j}=0}$$

Note:

$$Q_{i,j-\frac{1}{2}} = \phi_{y_{i,j-\frac{1}{2}}} = V_\infty\left(\frac{dy}{dx}\right)_{body} = \left(\frac{dy}{dx}\right)_{body}$$

If velocities normalized so that $V_\infty = 1$

At a point such as B, where only the c_p distribution is known, φ is prescribed from the relationship

$$\varphi_B = \varphi_A + \overbrace{\int_A^B \varphi_x \, dx}^{(d\varphi)_{AB}}$$

Where:

$\varphi \equiv$ Perturbation Velocity Potential

Pressure Coefficient: $\boxed{C_p \equiv \dfrac{P - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \dfrac{P - P_\infty}{\frac{\gamma}{2} P_\infty M_\infty^2}}$

Dynamic Pressure (show, at home, that these are equal)

$$C_p = \frac{P/P_\infty - 1}{\frac{\gamma}{2} M_\infty^2}$$

Recall our isentropic relation:

$$P = P_\infty \left[1 - \frac{\gamma-1}{2} M_\infty^2 \left(\frac{V^2}{V_\infty^2} - 1\right)\right]^{\frac{\gamma}{\gamma-1}}$$

Combine these last two expression (and see next page) to show that...

$$\boxed{\text{For Small Disturbances: } C_p \cong -\frac{2\varphi_x}{V_\infty}}$$

• Thus the φ_B equation above becomes

$$\varphi_B = \varphi_A + \int_A^B \left(-\frac{V_\infty C_p}{2}\right) dx \Rightarrow \boxed{\varphi_B = \varphi_A - \frac{V_\infty}{2} \int_A^B C_p \, dx}$$

If non-dimensionalize so $V_\infty = 1$, then $\varphi_B = \varphi_A - \frac{1}{2}\int_A^B C_p \, dx$

3.6 Small Disturbance Approximation for Surface pressure Coefficient., C_p:

We next develop a small disturbance approximation for the surface pressure. Our starting point is equations (3.2) and (3.4), which may be combined to yield:

$$C_p = \frac{\left[1+\frac{\gamma-1}{2}M_\infty^2\left(1-\frac{u^2+v^2}{V_\infty^2}\right)\right]^{\frac{\gamma}{\gamma-1}}-1}{\frac{\gamma}{2}M_\infty^2}$$

$$= \frac{\left[1+\frac{\gamma-1}{2}M_\infty^2\left(1-\frac{(V_\infty+\varphi_x)^2+\varphi_y^2}{V_\infty^2}\right)\right]^{\frac{\gamma}{\gamma-1}}-1}{\frac{\gamma}{2}M_\infty^2}$$

$$\approx \frac{\left[1+\frac{\gamma-1}{2}M_\infty^2\left(-2\frac{\varphi_x}{V_\infty}\right)\right]^{\frac{\gamma}{\gamma-1}}-1}{\frac{\gamma}{2}M_\infty^2}$$

In arriving at the above form, we have neglected second powers of terms such as φ_x/V_∞ and φ_y/V_∞.

Next, we use the binomial expansion

$$(1+\varepsilon)^n \approx 1+n\varepsilon \tag{3.20}$$

where ε is any small quantity $\varepsilon \ll 1$ and n is any real number. Then,

$$\cancel{C_p \approx \frac{\left[1-\frac{\gamma}{2}M_\infty^2\varphi_x-1\right]}{\frac{\gamma}{2}M_\infty^2}} \xrightarrow{\text{Correction}} C_p \approx \frac{\left[1+\frac{\gamma}{2}M_\infty^2\left(-2\frac{\varphi_x}{V_\infty}\right)\right]-1}{\frac{\gamma}{2}M_\infty^2}$$

$$\boxed{C_p \approx -2\frac{\varphi_x}{V_\infty}} \Leftarrow \text{For small disturbance flow}$$

where A is the aftmost point of the airfoil where slope is prescribed. Since φ is explicitly know at point B, and all the downstream points C, D, etc., the governing equation is not solved here. The SLOR procedure along the y-lines containing points B, C, D, etc. is applied between J=3 and JMAX-1.

After each relaxation sweep, φ_Z is known from the latest sweep. Therefore, φ_B, φ_C, etc. may be computed. The relaxation process is repeated several hundred time until φ everywhere ceases to change.

After the relaxation process has converged, dy/dx values ~~ore~~ are found at the solid surface immediately beneath the points B, C, D, etc. from the φ field using the relationship

$$\boxed{\frac{dy}{dx} = \varphi_y\Big|_{i,1\frac{1}{2}}}$$

Conversely, these values may be found from the TSD equation re-written as

$$\boxed{\left.\frac{dy}{dx}\right|_i = Q_{i,j+\frac{1}{2}} + \frac{\Delta y_{j+1} + \Delta y_j}{\Delta x_{i+1} + \Delta x_i}\left[\left(1-\mu_{ij}\right)\left(P_{i+\frac{1}{2},j} - P_{i-\frac{1}{2},j}\right) + \mu_{ij}\left(P_{i-\frac{1}{2},j} - P_{i-\frac{3}{2},j}\right)\right]}$$

Knowing dy/dx everywhere, the airfoil is computed as

$$\boxed{y(x) = y_A + \int_A^x \left(\frac{dy}{dx}\right) dx}$$

Carlson's technique is extremely simple. It may be incorporated into any TSD or FPE program. It may easily be extended to a 3-D wing design as well.

In Carlson's approach, when the final airfoil shape is obtained, the trailing edge occasionally remains open or has a cross-over creating a fish-tail. The nose region must be opened up more, or pressed down, and the design performed again until the trailing edge closes properly.

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