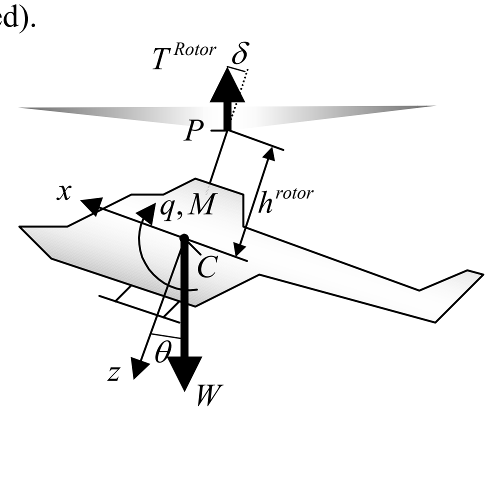
AE6520
Fall 2003
Homework #7
Due: Tuesday December 2, 2003 at 12:05pm (beginning of class)

1. Linearize the pitching and yawing moment equations (equations for $\dot{q}$ and $\dot{r}$) about a quasi-steady rolling equilibrium flight ($p \neq 0$, $q = 0$, and $r = 0$) for an axi-symmetric aircraft (symmetric about the x-axis).

2. A ball is balanced on a vertical spreading jet of air. The speed of the jet decreases linearly with height given by $V = V_0 - V_1 h$ where V_0 and V_1 are positive constants. The air mass density in the jet equals ρ, and c_D of the ball is based on its projected area, A. The ball has a mass of m. When released, the ball positions itself at a height where the weight and drag balance. Write an equation governing the vertical motion of the ball when perturbed from its equilibrium height and solve for the period of motion assuming the damping is less than critical.

3. Consider the point mass approximation of an aircraft with is constrained to move only in the vertical plane with its thrust along the velocity vector.
   (a) Linearize the resulting equations of motion about V_e, γ_e (where V is vehicle speed and γ is flight path angle) by considering small perturbations of these two parameters assuming that c_L is constant and $T - D = m\left(a_0 + a_1 V^2\right)$ where a_0 and a_1 are constants.
   (b) Obtain conditions on a_0 and a_1 for local stability of level flight at a constant altitude.

4. Consider the pitching motion in a plane of an idealized helicopter, shown in the figure. Only two external forces act on the helicopter. The first force is the thrust of the rotor T^{Rotor}, which acts at a point *P* which is h^{Rotor} above the center of mass, *C*, and is tilted forward with respect to the body as shown in the figure by angle δ. The second external force is gravity, *W*, effectively acting at the center of mass. The velocity of point *P* is $\begin{bmatrix} u & v & w \end{bmatrix}^T$. The mass of the helicopter is *m*, the vehicle is symmetric about the *Cxz* plane and the inertia about point *C* around the y-axis is I_y. The machine is restricted to move in a plane, so $\phi = \psi = p = r = v = 0$ (helicopter can change pitch attitude, move for/aft, and up/down – but no other motion is allowed).



The body x-force equation, the body z-force equation, the pitch moment equation, and the pitch attitude kinematics, where the origin of the body frame is point P, has been shown to be:

$$T^{Rotor} \sin\delta - W \sin\theta = m\left(\dot{u} + qw + \dot{q}h^{Rotor}\right)$$
$$-T^{Rotor} \cos\delta + W \cos\theta = m\left(\dot{w} - qu - q^2 h^{Rotor}\right)$$
$$-Wh^{Rotor} \sin\theta = mh^{Rotor}\left(\dot{u} + qw\right) + \left[I_y + m\left(h^{Rotor}\right)^2\right]\dot{q}$$
$$\dot{\theta} = q$$

Linearize these equations of motion about hover equilibrium flight $\theta_e = q_e = u_e = w_e = 0$. Remember that δ and T^{Rotor} are going to vary with flight condition (u, w, q). For the purposes of this problem, any controls remain fixed at equilibrium positions.

4. A hovercraft in ground effect is (approximately) acted on by the following *aerodynamic* forces and moments, expressed in the body frame:

$$X = Y = 0, Z = Z(h)$$
$$L = L(\phi), M = M(\theta), N = 0$$

With the dependency of z-force, rolling and pitching moment on vehicle state as indicated. Here, h is the vertical position (altitude) with respect to the Earth, positive up. The rolling/pitching moment is zero when the roll/pitch angle is zero respectively. The z-axis aerodynamic force opposes gravity at the equilibrium flight condition. The body axes are principal axes. The engine/rotor angular momentum expressed in the body frame is:

$$\mathbf{h}_r^b = \begin{bmatrix} 0 & 0 & H \end{bmatrix}^T$$

(a) Derive a set of small-disturbance equations of motion.
(b) Find the characteristic equation of this hovercraft.
(c) Show that when it is statically *un*stable (with both $\frac{\partial M}{\partial \theta}$ and $\frac{\partial L}{\partial \phi}$ positive) it can be gyrostabilized (like a spinning top) if H is large enough. hint: showing modes have real parts less than *or equal* to zero will be sufficient.