AE6520
Fall 2003
Homework #3
Due: Thursday October 9, 2003 at 12:05pm (beginning of class)

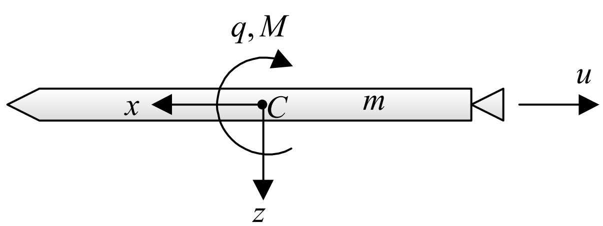
1. Problem 1.2-8 in your text.

2. A new type of solid fuel rocket is initially spinning at an angular rate of $p_o$ about the body x axis. The inertia about that axis is $mk_x^2$ where $m$ is the mass of the rocket. The rocket exhaust is located precisely on the vehicle body x-axis. The radius of gyration, $k_x$, and the location of the center of mass remain constant as the rocket burns down to 10% of the original mass. The rate of change of mass is a constant $\dot{m} < 0$. Ignore pitch and yaw coupling, and neglect any external torques.

(a) If the initial mass was $m_o$, then what is $p$ at burnout (use conservation of angular momentum)?

(b) Derive an expression for $p(t)$.

3. Consider the solid-fueled rocket shown in the figure. Given that the pitch moment of inertia about the center of mass (point $C$) can be expressed as $I_y = mk_y^2$ where $k_y$ is the current radius of gyration about the y-axis and $m$ is the current mass of the rocket (both $m$ and $k_y$ vary with time in this problem). The exhaust velocity is directed along the body axes unit vector $\begin{bmatrix} e_x & e_y & e_z \end{bmatrix}^T$, with exit speed $u$. The rocket exhaust is located in the body axes at $\begin{bmatrix} x_e & y_e & z_e \end{bmatrix}^T$.



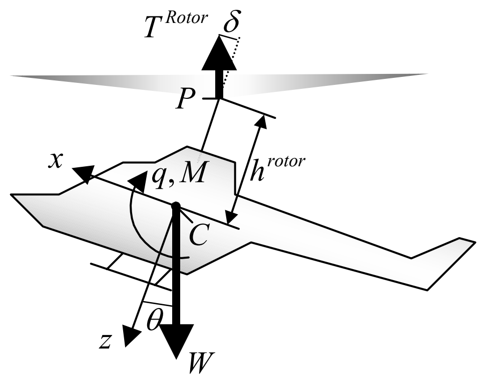

(a) Derive an expression for the thrust of the rocket.

(b) Derive an expression for the moment due to the rocket motor about $C$.

(c) Assuming body axis roll and yaw angular rates are zero, show that the pitch-moment equation is $M = I_y\dot{q} + q\left(m\frac{d(k^2)}{dt} + k^2\dot{m}\right) - \dot{m}\left[(x_e)^2 + (z_e)^2\right]q$ where $q$ is the pitch rate and $M$ is the sum of external and rocket moments about the body y-axis.

4. Consider the pitching motion in a plane of an idealized helicopter, shown in the figure below. Only two external forces act on the helicopter. The first force is the thrust of the rotor $T^{Rotor}$, which acts at a point $P$ which is $h^{Rotor}$ above the center of mass, $C$, and is tilted forward with respect to the body as shown in the figure by angle $\delta$. The second external force is gravity, weight $W$, effectively acting at the center of mass. The velocity of point $C$ is $\begin{bmatrix} u & v & w \end{bmatrix}^T$ in the body frame, for point $P$ it is $\begin{bmatrix} u_P & v_P & w_P \end{bmatrix}^T$. The mass of the helicopter is $m$, the vehicle

is symmetric about the *Cxz* plane and the inertia about point C around the y-axis is $I_y$. The machine is restricted to move in a plane, so $\phi = \psi = p = r = v = v_P = 0$ (helicopter can change pitch attitude, move for/aft, and up/down – but no other motion is allowed) and treated as a rigid body.




(a) We are going to develop equations of motion to analyze this idealized helicopter for studies of take-off and landing. Is it reasonable to neglect the rotation and/or curvature of the Earth for this problem? Why or why not?

(b) Derive the pitch moment equation, body x-force equation, and the body z-force equation about point C, with external forces and moments specified as described above (function of $T^{Rotor}$, *W*, etc.).

(c) Derive the pitch moment equation, body x-force equation, and the body z-force equation again, where this time the origin of the body frame has been relocated to point P (and so it will be in terms of $u_P, w_P$ rather than $u, w$).