

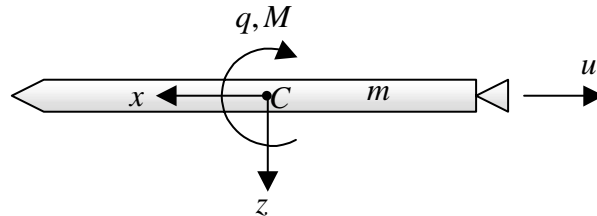
AE6520
Fall 2002
Homework #4

Due: Friday October 4, 2002 at 11am (beginning of class)

- A new type of solid fuel rocket is initially spinning at an angular rate of p_o about the body x axis. The inertia about that axis is mk_x^2 where m is the mass of the rocket. The rocket exhaust is located precisely on the vehicle x axis. The radius of gyration, k_x , and the location of the center of mass remain constant as the rocket burns down to 10% of the original mass. The rate of change of mass is a constant $\dot{m} < 0$. Ignore pitch and yaw coupling, and neglect any external torques.

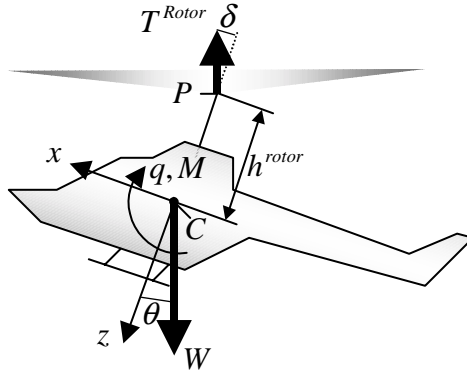
 - If the initial mass was m_o , then what is p at burnout (use conservation of angular momentum)?
 - Derive an expression for $p(t)$.

- Consider the solid-fueled rocket shown in the figure. Given that the pitch moment of inertia about the center of mass (point C) can be expressed as $I_y = mk_y^2$ where k_y is the current radius of gyration about the y -axis and m is the current mass of the rocket (both m and k_y vary with time in this problem). The exhaust velocity is directed along the body axes unit vector $[e_x \ e_y \ e_z]^T$, with speed u . The rocket exhaust is located in the body at $[x^e \ y^e \ z^e]^T$.



- Derive an expression for the thrust of the rocket.
 - Derive an expression for the moment of the rocket about C.
- (c) Show that the pitch-moment equation is $M = I_y \dot{q} + q \left(m \frac{d(k^2)}{dt} + k^2 \dot{m} \right) - \dot{m} \left[(x^e)^2 + (z^e)^2 \right] q$ where q is the pitch rate and M is the sum of external and rocket moments about the body y -axis.
- Consider the pitching motion in a plane of an idealized helicopter, shown in the figure. Only two external forces act on the helicopter. The first force is the thrust of the rotor T^{Rotor} , which acts at a point P which is h^{Rotor} above the center of mass, C, and is tilted forward with respect to the body as shown in the figure by angle δ . The second external force is gravity, weight W , effectively acting at the center of mass. The velocity of point C is $[u \ v \ w]^T$ in the body frame, for point P it is $[u^P \ v^P \ w^P]^T$. The mass of the helicopter is m , the vehicle

is symmetric about the Cxz plane and the inertia about point C around the y-axis is I_y . The machine is restricted to move in a plane, so $\phi = \psi = p = r = v = v^P = 0$ (helicopter can change pitch attitude, move for/aft, and up/down – but no other motion is allowed) and treated as a rigid body.



- We are going to develop equations of motion to analyze this idealized helicopter for studies of take-off and landing. Is it reasonable to neglect the rotation and/or curvature of the Earth for this problem? Why or why not?
- Derive the pitch moment equation, body x-force equation, and the body z-force equation about point C, with external forces and moments specified as described above (function of T^{Rotor} , W , etc.).
- Derive the pitch moment equation, body x-force equation, and the body z-force equation again, where this time the origin of the body frame has been relocated to point P (and so it will be in terms of u^P, w^P rather than u, w).