

AE6520
Fall 2002
Homework #2

Due: Monday September 16, 2002 at 11am (beginning of class)

1. (a) Derive this variation on Equation (4.4,7) in your text, page 100:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi \tan\theta & \cos\phi \tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi \sec\theta & \cos\phi \sec\theta \end{bmatrix} (\omega_B - \omega_B^V)$$

where ω_B is the angular velocity of the body frame with respect to the inertial frame, expressed in the body frame and ω_B^V is the angular velocity of the vehicle-carried frame with respect to the inertial frame, also expressed in the body frame. Note that the version in the text neglects ω_B^V (which is perfectly reasonable for many problems).

- (b) Using the definition of the F_V frame, under what conditions is ω_B^V going to be negligible?

2. Quaternion questions

- (a) Given $\phi = 0$, $\theta = 0$, and $\psi = \pi$, about what unit vector could one modify the F_V frame to get the F_B frame with a single rotation? What is the single angle (θ') of rotation? What are the components of the quaternion corresponding to this single rotation

$$(q_{BV} = [q_1 \ q_2 \ q_3 \ q_4]^T)?$$

- (b) Same questions as (a), but this time $\phi = \pi$, $\theta = 0$, and $\psi = -\frac{\pi}{2}$.

3. Given $q_{BV} = [q_1 \ q_2 \ q_3 \ q_4]^T$, derive a general expression for ϕ , θ , and ψ (Hint: utilize your answer from a question on HW#1).

4. The new improved Space Shuttle Orbiter is on a special mission, where it has achieved an equatorial orbit, and has maneuvered such that body attitude remains fixed compared with distant stars. An experimental new navigation system has precisely determined the current vehicle center of mass position with respect to the center of the Earth, along with derivatives:

$$r_{ECEF} = \begin{bmatrix} 2.2 \times 10^7 \\ 0 \\ 0 \end{bmatrix} ft, \quad \dot{r}_{ECEF} = \begin{bmatrix} 0 \\ 2.37 \times 10^4 \\ 0 \end{bmatrix} \frac{ft}{sec}, \quad \text{and} \quad \ddot{r}_{ECEF} = \begin{bmatrix} -25.6 \\ 0 \\ 0 \end{bmatrix} \frac{ft}{sec^2}.$$

Assume that the magnitude of the angular velocity of the Earth with respect to F_{ECI} is 0.000073 rad/sec , and that the total mass of the Orbiter is $220,000 \text{ lb}$.

- (a) What is the velocity of the orbiter center of mass with respect to F_{ECI} (expressed in F_{ECEF})? What is the acceleration of the orbiter center of mass with respect to F_{ECI} (expressed in F_{ECEF})?
- (b) What is the angular momentum of the Orbiter about the center of the Earth (i.e., cross product of position and linear momentum)?