Homework Assignment #6 AE-3521
Date: 2/11/04 Due: 2/23/04

This assignment counts as a double assignment. Problems 5 and 7 are optional and will be counted as extra credit.

1. Problem 4.14. Straightforward application of Eqs. (4.47, 4.48). The coefficient in (4.47) follows from the fact that $e^{-0.693} = ½$, and of course N = time to half amplitude / period, so the coefficient in (4.48) is $0.693/(2\pi) = .110$. I think it is better to remember how these expressions are obtained, rather than memorize the expressions themselves.

2. Problem 4.16. Start with the approximate expressions for natural frequency and damping ratio for the short period and phugoid modes. Replace density ρ in the stability derivative expressions in Table 4.2 by $\sigma\rho_o$ where ρ_o is the sea-level density. Then find the dependence of ω_n and ζ on σ and their corresponding sea-level values for these modes. Answer: $\sqrt{\sigma}\ x\,(\mathit{corresponding\ sealevel\ value})$

3. Problem 4.17. Set u=q=θ=0 in the linearized longitudinal equations of motion, and determine the eigenvalue associated with the resulting first order differential equation of motion. Also, answer the following questions. What derivative does it depend on? What sign on this derivative will insure stability? Assuming the derivative has the correct sign, then what is the corresponding 5% settling time?

4. Problem 4.19, part c) using an approximate model for the short period and phugoid modes.

5. Problem 4.22. Use the data to estimate $\eta = -\zeta\omega_n$ and $\omega = \omega_n\sqrt{1-\zeta^2}$. Recall that the period of oscillation = $2\pi/\omega$. However, what is more important in this problem is that you realize where the expressions that relate the stability derivatives to these quantities come from. Start with $I_y\dot{q} = M$. Next substitute a linear aero model for M by specializing the 2nd expression in (3.49) to the case $\Delta u = \Delta\delta_e = \Delta\delta_T = 0$. Using α in place of Δw (recall that $\alpha = \Delta w/u_o$) we get $M/I_y = M_\alpha\alpha + M_{\dot{\alpha}}\dot{\alpha} + M_q q$. Now recognize that when the motion is constrained to a single degree of freedom in pitch, then $a = \theta$ and $q = \dot{\theta} = \dot{\alpha}$. Therefore, it follows that $I_y\ddot{\theta} - (M_{\dot{\alpha}} + M_q)\dot{\theta} - M_\alpha\theta = 0$, from which it can be seen that $2\zeta\omega_n = -2\eta = -(M_{\dot{\alpha}} + M_q)$ and $\omega_n = \sqrt{-M_\alpha}$. So the expression $\omega = \sqrt{-M_\alpha}$ is approximate, and assumes that $\zeta << 1$. A better approach is to not introduce this approximation, and estimate M_α using $\omega_n = \sqrt{\omega^2 - \eta^2} = \sqrt{-M_\alpha}$.

6. Problem 4.18. Before doing this problem check that the signs of all the stability and control derivative terms for this aircraft are correct, based on what you have learned thus far concerning what the sign should be. You answer for the longitudinal modes should closely agree with the values given below. It's important you know how to do this so you don't waste a lot of effort using incorrect data, and it will also be useful for the final exam.

In doing this problem I expect you to use the notes on "Longitudinal Analysis of the A4D Aircraft" as a guide to do the following:

a) Calculate the natural frequency and damping ratios for the short period and phugoid modes, and estimate $N_{1/2}$ or N_{double}, and $T_{1/2}$ or T_{double} for both modes. Also do this using the phugoid and short period approximations. See partial answer below. *Note that the phugoid eigenvalues are real at this flight condition.*

b) Draw the Argand diagrams for each mode. Exhibit the individual modal responses on times scales that are appropriate for each mode. In particular show that the time responses are consistent with what is predicted by the Argand diagrams in terms of relative magnitudes and phases by measuring these quantities directly off of your plots.

c) Exhibit both the short term and long term state responses for a 3 second elevator pulse input having a magnitude of 0.1 rad.

d) Show the Bode plot for the transfer function $G_{\alpha,\delta e}(s)$, and discuss any near pole/zero cancellations you observe in this transfer function. By canceling these poles and zeros, develop a second order transfer function approximation made up of one zero and two poles in the form

$$G_{\alpha,\delta_e}(s) \approx \frac{K(s/z+1)}{(s/\omega_n)^2 + 2\zeta s/\omega_n + 1}$$

where K is chosen so that at s=0 the approximation matches that of the full order transfer function. Overlay a Bode plot of this transfer function with that of the original to evaluate your approximation in the frequency domain. To overlay a plot in matlab, enter the command ‘hold’ after getting the first plot and leave this figure active in you window.

e) Simulate the α response to a sinusoidal elevator input, $\delta_e = 0.2 \sin(\omega t)$, for $\omega = 0.5\ \omega 3db$, $\omega = \omega 3db$, and $\omega = 2.0\ \omega 3db$, where $\omega 3db$ is the frequency in the Bode plot of part d) where the magnitude plot is 3 db down from its low frequency value. Compare the magnitudes and phase shifts you observe with those predicted from the Bode plot in part d)

f) Prepare a report in the style of my notes on Longitudinal Analysis summarizing your problem 3 results.

Jetstar Longitudinal Eigenvalues (M=0.8, h=40,000 ft)

-7.8082e-001 ± 3.1227e+000 i
-3.0307e-003 ± 4.1966e-002 i

7. Also, if you would like to try some problems in Chap 5, take a look at 5.2, 5.3 and 5.6. In 5.2, use the general solution in (5.5) for a step response for the roll mode approximation $\dot{p} = L_p p + L_{\delta_a} \delta_a$. See also example 5.1 on page 183. In 5.2, use the general solution for the response to an initial condition $p(t) = p(0)e^{-t/\tau}$, $\tau = -1/L_p$. In 5.6, for a flat plate, the theoretical value of C_{l_β} is 2*pi, which should be corrected for a finite aspect ratio using the expression on p. 57. Note that $\psi = -\beta$ for the single degree of freedom model. Since there are no wings, one can neglect $N_{\dot{\beta}}$, and retain only the N_r and N_β. Therefore, $\ddot{\psi} - N_r\dot{\psi} - N_\beta\psi = 0$ where $N_r = C_{n_r}(\frac{D}{2u_o})QSD/I_z$, and $N_\beta = C_{n_\beta}QSD/I_z$. To estimate the aero coefficients, use $C_{n_\beta} = \eta V_v C_{l_\beta}(1 + \frac{d\sigma}{d\beta})$, where $V_v = \frac{l_v S_v}{SD}$, with $S_v = 3D^2$ Sv=3D*D, $l_v 5.25D$ (measured from the ¼ chord point location of the aerodynamic center of the tail fin. Note that $\frac{d\sigma}{d\beta} = 0$ (since there are no wings) and as an approximation use $\eta = 1$.