

## Homework Assignment #4

Date: 1/26/04

AE-3521

Due: 2/02/04

1. Estimate  $C_{m_{\dot{\alpha}}}$  and  $C_{m_q}$  in Problem 3.5.
2. Problem 3.6
3. Estimate  $C_{n_r}$  in Problem 3.9, and compare to the value in Appendix B.
4. Problem 3.12
5. A reentry vehicle is entering the Earth's atmosphere at a latitude of 38 degrees. It's inertial velocity is 20,000 ft/s at a heading of 45 degrees, The flight path angle nearly tangential to the Earth's surface, so for this problem you can approximate it as zero.
  - a) Calculate the vehicle's speed and heading relative to the Earth's surface.
  - b) The on-board accelerometers read  $[1.5; 0; 0]$  g's. Calculate the acceleration relative to the Earth's surface. *Note* that this result is needed to navigate relative to the Earth's surface using on-board sensors. That is, given accelerometer and gyro readings, we can calculate  $\ddot{\vec{r}}(t)$ , and then integrate twice to obtain  $\vec{r}(t)$ . The same idea can be used to navigate relative to any planetary body.

### Hints and answers:

1. Use the expressions in Table 3.3. Assume  $\eta=1$ . Note that  $\bar{c} = S/b$ , and use (2.22) to estimate  $d\epsilon/d\alpha$ , where  $AR_w = b/\bar{c} = b^2/S$ . The theoretical value of  $C_{l_{\alpha_w}} = 0.1$  needs to be corrected for a wing with finite aspect ratio using the expression on page 57 (note the units are /rad). The same goes for the tail surfaces. The tail volume ratio,  $V_H$ , is defined on page 48, where  $S_t$  is the reference area for the horizontal tail. Ans: -10.4 and -19.7.
2. Use the expressions in Table 3.4, again correcting  $C_{l_{\alpha_w}}$  for finite aspect ratio. The wing tip chord and root chord have to be estimated from the figure. Use  $c_t = 4.4$  ft. and  $c_r = 9$  ft. The aspect ratio for the vertical fin needs to be estimated from the figure. To do this, visualize the vertical fin as a wing, whose span ( $b_v$ ) is twice the height of the fin. Use 6.5 ft for the height. Then  $AR_v = \frac{b_v^2}{2S_v}$ . Assume  $\eta_v = 1$ . Ans: -0.6 and 0.27
3. From the drawing and the data given on page 417, use the following estimates:  $S=5500$  ft<sup>2</sup>,  $S_v = 900$  ft<sup>2</sup>,  $b_v/2 = 32.6$  ft and  $l_v = 105$  ft to get  $V_v$  and  $AR_v$ , and estimate  $C_{L_{\alpha_v}}$  using the expression on page 57. Assume  $\eta_v = 1$ . Ans: -0.3
5. Solve this problem in a rotating Earth fixed frame, with the x and y axes in the equator, the z axis passing through the North pole, and the entry condition located in the x,z plane. In part a) use  $\vec{V} = \dot{\vec{r}} + \vec{\omega} \times \vec{r}$ . From the entry conditions given, you should be able to express the inertial velocity ( $\vec{V}$ ) and position ( $\vec{r}$ ) in the Earth fixed frame, and then solve for  $\dot{\vec{r}}$ . In part b) use  $\vec{a} = \ddot{\vec{r}} + 2\vec{\omega} \times \dot{\vec{r}} + \vec{\omega} \times \vec{\omega} \times \vec{r} = \vec{F}/m$  and solve for  $\ddot{\vec{r}}$ , using  $\dot{\vec{r}}$  from part a) and calculating  $\vec{F}/m$  from the accelerometer readings. Remember that accelerometers sense

only the specific force, (aero + propulsive force)/mass, so you need to add in the gravity term.

**Answers for problems 1 to 3 are from the solutions manual, and are not guaranteed. Let me know if your answer is different. Be careful of units. Your result should be dimensionless, or equivalently, /rad.**