AE 3521 HW #8 Solutions

1)

$$t_{half} = (\text{time when } p = 5 \tfrac{rad}{sec})$$

$$= 1.6 \text{ sec}$$

$$t_{half} = \frac{\log(2)}{|\lambda|}$$

$$\lambda_R = -0.43$$

$$L_p \cong \lambda_R = -0.43$$

$$\boxed{L_p = -0.43}$$

2) $\lambda_{sp} = -0.5$

$\lambda_R = -7.0$

$\lambda_{DR} = -2 \pm 3i$

$$T_{1/2\ sp} = \frac{\log(2)}{|\lambda_{sp}|} = \boxed{1.38 \text{ sec}}$$

$$T_{1/2\ R} = \frac{\log(2)}{|\lambda_R|} = \boxed{0.10 \text{ sec}}$$

$$T_{1/2\ DR} = \frac{\log(2)}{|Re(\lambda_{DR})|} = \frac{\log(2)}{|-2|} = \boxed{0.345 \text{ sec}}$$

$$Period_{DR} = \frac{2\pi}{\omega} = \frac{2\pi}{3} = \boxed{2.09 \text{ sec}}$$

3) $\dot{x} = Ax + B\eta$

$$A = \begin{bmatrix} -0.0506 & -0.984 \\ 0.64 & -0.34 \end{bmatrix} \qquad B = \begin{bmatrix} 0.034 \\ -0.616 \end{bmatrix}$$

(a) $\det(\lambda I - A) = 0$

$$\lambda^2 + 0.39\lambda + 0.647 = 0$$

$$\boxed{\lambda = -0.195 \pm 0.78\,i}$$

(b) $\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$

$$\omega_n^2 = 0.647$$

$$\boxed{\omega_n = 0.8 \ \frac{rad}{sec}}$$

$$2\zeta\omega_n = 0.39$$

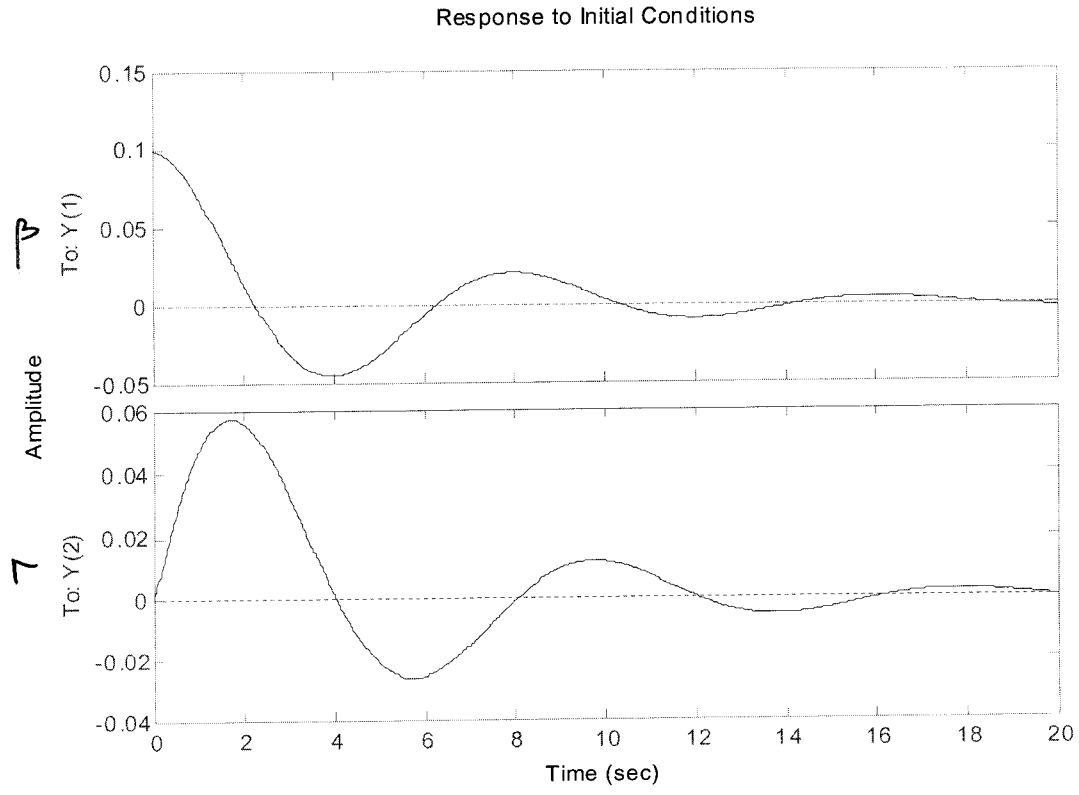
$$\boxed{\zeta = 0.244}$$

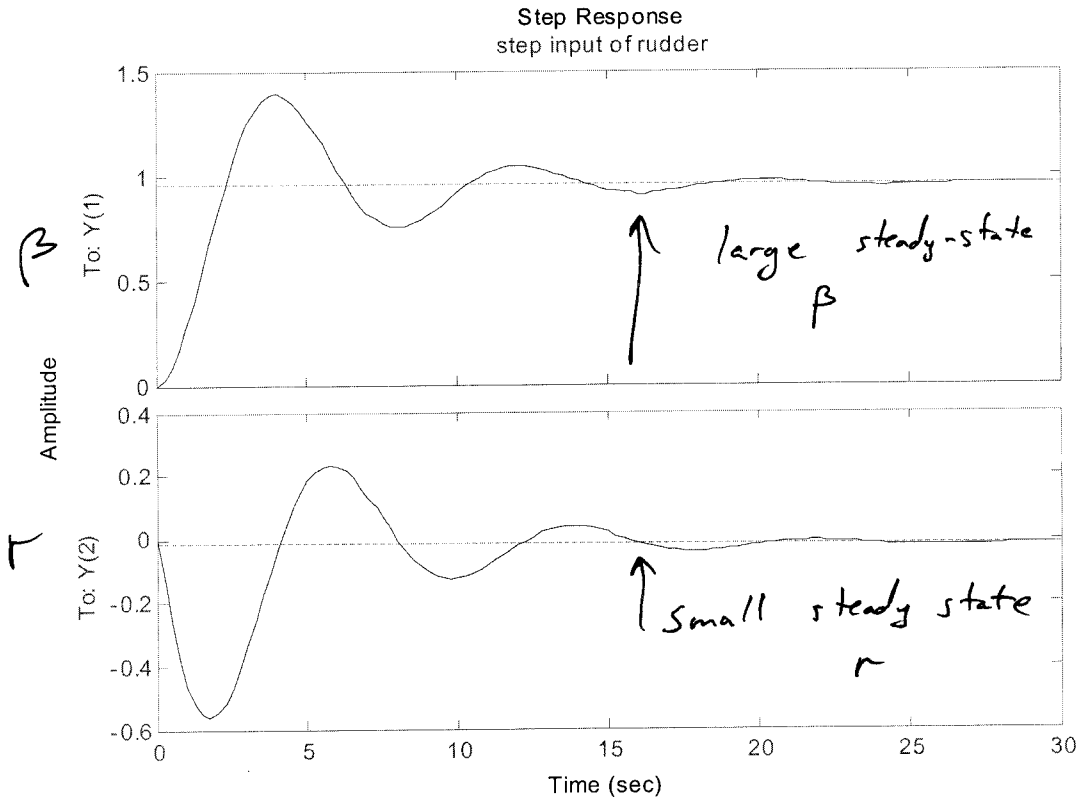
(c) $t_{1/2} = \dfrac{\log(2)}{\zeta\omega_n} = \dfrac{0.69}{0.195} = \boxed{3.54 \text{ sec}}$

$$\text{period} = \frac{2\pi}{\omega} = \frac{2\pi}{0.78} = \boxed{8.05 \text{ sec}}$$

parts d and e) Matlab file:

```
Ybeta = -7.8;
Yr = 2.47;
Nbeta = 0.64;
Nr = -0.34;
u0 = 154;
Ydr = 5.236;
Ndr = -0.616;

A=[Ybeta/u0 -1+Yr/u0;Nbeta Nr];
B=[Ydr/u0;Ndr];
C=eye(2);
D=[0;0];

% part d

x0 = [0.1;0];
t=0:.025:20;
initial(A,B,C,D,x0,t);
title('initial condition')
pause;

% part e

step(A,B,C,D);
title('step input of rudder')
```




Step Response
step input of rudder
Amplitude
To: Y(1)
To: Y(2)
1.5
1
0.5
0
0.4
0.2
0
-0.2
-0.4
-0.6
0
5
10
15
20
25
30
Time (sec)
β
r
large steady-state β
small steady state r