AE3521 HW#3 Solutions

1) $\bar{A} = \frac{{}^i d}{dt} \bar{V}$

$= \frac{{}^b d}{dt} \bar{V} + \bar{\omega}^{bi} \times \bar{V}$

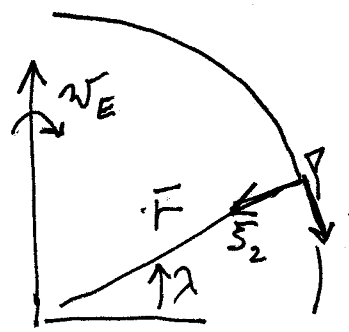
$\bar{\omega}^{bi} = \dot{\gamma} \bar{b}_3$

$\bar{V} = V \bar{b}_1 \implies \frac{{}^b d}{dt} \bar{V} = \dot{V} \bar{b}_1$

$\bar{A} = \dot{V} \bar{b}_1 + \dot{\gamma} \bar{b}_3 \times V \bar{b}_1$

$$\boxed{\bar{A} = \dot{V} \bar{b}_1 + V\dot{\gamma} \bar{b}_2}$$

2)



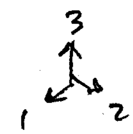
$\bar{s}$ is Earth fixed
(A/C flying past)

$\bar{s}_3$ into page

Coriolis accel $= \bar{a} = 2\,\bar{\omega}^{si} \times \frac{{}^s d}{dt}\bar{r}$

$$\bar{\omega}^{si} = \omega_E(-\bar{s}_1 \cos\lambda - \bar{s}_2 \sin\lambda)$$

$$\frac{{}^s d}{dt}\bar{r} = V\bar{s}_1$$



$$\bar{a} = 2(-\omega_E \overset{\sin}{\cancel{\cos}}\lambda\, \bar{s}_2) \times V\bar{s}_1$$

$$= 2\,\omega_E \overset{\sin}{\cancel{\cos}}\lambda\, V\, \bar{s}_3$$

$$\boxed{\bar{a} = 2\,\omega_E \sin\lambda\, V\, \bar{s}_3}$$

(East)

if regarded as a force, we would be pushed to the right in the northern hemisphere, left in the southern

3) 1.6) $\bar{r} = x\bar{s}_1 + y\bar{s}_2 \qquad \dot{w} = 0$

$$\frac{\bar{F}}{m} = \frac{{}^i d^2}{dt^2}\bar{r} = \frac{{}^s d^2}{dt^2}\bar{r} + 2\bar{w}^{si} \times \frac{{}^s d}{dt}\bar{r} + \bar{w}^{si} \times (\bar{w}^{si} \times \bar{r})$$

$$\frac{\bar{F}}{m} = (\ddot{x}\bar{s}_1 + \ddot{y}\bar{s}_2) + 2(w\bar{s}_3) \times (\dot{x}\bar{s}_1 + \dot{y}\bar{s}_2)$$
$$+ (w\bar{s}_3) \times (w\bar{s}_3 \times (x\bar{s}_1 + y\bar{s}_2))$$

$$\frac{\bar{F}}{m} = \ddot{x}\bar{s}_1 + \ddot{y}\bar{s}_2$$
$$+ (-2w\dot{y}\bar{s}_1 + 2w\dot{x}\bar{s}_2)$$
$$+ (-w^2 x\bar{s}_1 - w^2 y\bar{s}_2)$$

$$\boxed{\begin{aligned} \bar{F}\circ\bar{s}_1 &= m(\ddot{x} - 2w\dot{y} - w^2 x) \\ \bar{F}\circ\bar{s}_2 &= m(\ddot{y} + 2w\dot{x} - w^2 y) \end{aligned}}$$

fall : $\bar{F} = 0$

initially, $y = 0,\ x > 0,\ \dot{y} = 0,\ \dot{x} > 0$

$$\ddot{y} + 2w\dot{x} - w^2 y = 0$$

$$\ddot{y} = \underbrace{-2w\dot{x}}_{<0} + \underbrace{w^2 y}_{\text{initially small}}$$

$\therefore$ y becomes ~~negative~~ negative

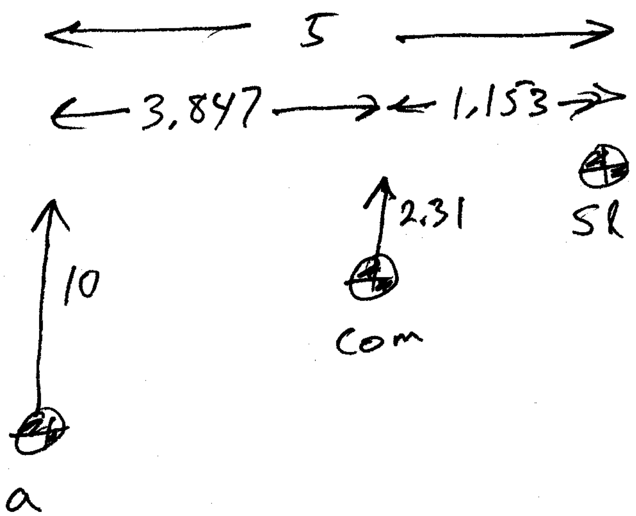
put ladder on "left" side $(-\bar{s}_2)$

4) (a)

$$\bar{V}_{com} = \frac{1}{m_a + m_{sb}} \left(m_a \bar{V}_a + m_{sl} \bar{V}_{sl} \right) =$$

$$= \frac{m_a}{m_a + m_{sl}} \bar{V}_a = \boxed{2.307 \, \bar{b}_2 \, \frac{m}{s}}$$

(b)



$\bar{V}_{sl}$ w/rt com $= -2.31 \, \bar{b}_2 \frac{m}{s}$

$\bar{V}_a$ w/rt com $= 7.693 \frac{m}{s} \bar{b}_2$

$$\bar{H} = \sum_{i=1}^{2} m_i \bar{r}_i \times \bar{V}_i$$

$$= -15000 \cdot 3.846 \cdot 7.693 \, \bar{b}_3 + -50000 \cdot 1.157 \cdot 2.307 \, \bar{b}_3 = \boxed{-5.772 \times 10^5 \, \frac{kg\,m^2}{sec} \, \bar{b}_3}$$

(c) Parallel Axis Theorem

$$I = I_{sl} + m_{sl}\begin{bmatrix} \Delta y^2+\Delta z^2 & -\Delta x\Delta y & \cdot \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$+ I_a + m_a\begin{bmatrix} \Delta y^2+\Delta z^2 & -\Delta x\Delta y & \cdot \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$I = I_{sl} + 50000\begin{bmatrix} .8519 & -1.065 & 0 \\ -1.065 & 1.331 & 0 \\ 0 & 0 & 2.183 \end{bmatrix}$$

$$+ I_A + \begin{bmatrix} 142 & -177.53 & 0 \\ -177.53 & 221.87 & 0 \\ 0 & 0 & 363.94 \end{bmatrix} \times 10^3$$

$$\boxed{I = \begin{bmatrix} 294.59 & -230.77 & 0 \\ -230.77 & 350.42 & 0 \\ 0 & 0 & 543.12 \end{bmatrix} \times 10^3 \text{ kgm}^2}$$

(d) $\bar{b}_3$ is a priciple axis of docked config, and $\bar{H}$ is in this direction

$\therefore \omega_1 = \omega_2 = 0$

$$\omega_3 I_z \bar{b}_3 = \bar{H} = 5.772 \times 10^5 \frac{kg m^2}{sec}$$

$$\boxed{|\omega_3| = \frac{|H|}{I_z} = 1.062 \frac{rad}{sec}}$$

$$\boxed{\omega_3 = -1.062 \frac{rad}{sec}}$$

Note: It can be shown this is a rotation about the intermediate moment of inertia

$\therefore$ ustable!