AE3521
Fall 2001 Homework #2
Due: Friday September 14, 2001 at noon (beginning of class) or before

1. The time derivatives of unit vectors $\vec{b}_i$ (which rotate with angular velocity $\vec{\omega}$) are found by:

$$\frac{d}{dt}\vec{b}_i = \vec{\omega}\times\vec{b}_i$$

Use this result to show:

$$\frac{{}^i d}{dt}\vec{v} = \frac{{}^b d}{dt}\vec{v} + \vec{\omega}^{bi}\times\vec{v}$$

where the angular velocity of the b frame with respect to the i frame is $\vec{\omega}^{bi}$.

2. Wiesel chapter 1, problem 5, p. 40.

3. Consider the torque-free rotational motion of the space shuttle orbiter that has lost attitude control. An inertia matrix is given for principal body axis in Wiesel p. 163 (axes illustrated in Figure 5.23 on the same page) as

$$I^b = \begin{bmatrix} 1.29 & 0 & 0 \\ 0 & 9.68 & 0 \\ 0 & 0 & 10.1 \end{bmatrix}\times 10^6 kg - m^2$$

(a) Given $0 = I^b\dot{\omega}^{bi} + \tilde{\omega}^{bi}\left(I^b\omega^{bi}\right)$, show body rotational dynamics can be described by:

$$\dot{\omega}_1 = -\frac{I_z - I_y}{I_x}\omega_2\omega_3,\ \dot{\omega}_2 = -\frac{I_x - I_z}{I_y}\omega_3\omega_1,\ \text{and}\ \dot{\omega}_3 = -\frac{I_y - I_x}{I_z}\omega_1\omega_2$$

(b) Use the attitude rotation angle order presented in class (this is similar to Wiesel pp. 114-115, but WARNING: he defines simple rotation matrices as the transpose of how I did it in class)

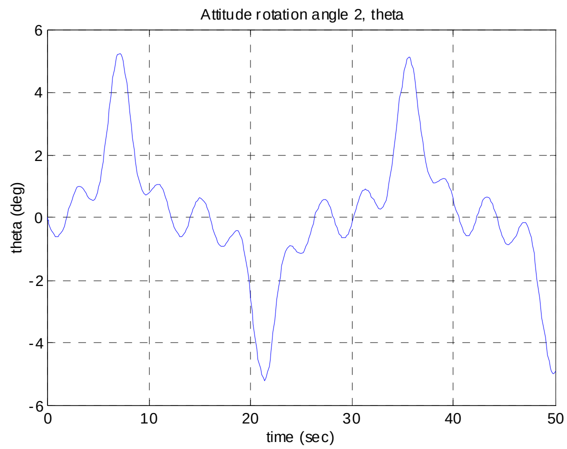
$$R^{ib} = R_1(\phi)R_2(\theta)R_3(\psi)$$

and show rotational kinematics are described by:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix}\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

(hint: you can show one matrix is the inverse of another by multiplying and getting an identity matrix)

(c) The orbiter is initially spinning at $\omega_1(0) = 0$, $\omega_2(0) = 90^o/\sec$, $\omega_3(0) = 1^o/\sec$ and has an attitude defined by $\phi(0) = 90^o$, $\theta(0) = 0^o$, and $\psi(0) = 0^o$. From (a) and (b), things are now in the form $\dot{x} = f(x)$. Use these results to do a simulation with these initial conditions out to 50 seconds. You my use MATLAB (using a command such as ode45), simulink, solve the exact analytic solution, or any other method you wish. You'll know things are going well when your plot of $\theta(t)$ looks like:



Include plots of all three angular rate components in the body frame and the three attitude rotation angles. Also include all source code, scripts, etc. that you used. (hint: our equations for dynamics only work when angular velocity is expressed in radians per second)